

Contents

<i>Preface</i>	ix
1 Multilinear mappings	1
General remarks	1
1.1 Multilinear mappings	1
1.2 The tensor notation	4
1.3 Tensor powers of a module	6
1.4 Alternating multilinear mappings	6
1.5 Symmetric multilinear mappings	10
1.6 Comments and exercises	13
1.7 Solutions to selected exercises	15
2 Some properties of tensor products	19
General remarks	19
2.1 Basic isomorphisms	19
2.2 Tensor products of homomorphisms	22
2.3 Tensor products and direct sums	25
2.4 Additional structure	28
2.5 Covariant extension	29
2.6 Comments and exercises	31
2.7 Solutions to selected exercises	37
3 Associative algebras	42
General remarks	42
3.1 Basic definitions	42
3.2 Tensor products of algebras	44
3.3 Graded algebras	47
3.4 A modified graded tensor product	51
3.5 Anticommutative algebras	54
3.6 Covariant extension of an algebra	56
3.7 Derivations and skew derivations	56

3.8	Comments and exercises	58
3.9	Solutions to selected exercises	65
4	The tensor algebra of a module	69
	General remarks	69
4.1	The tensor algebra	69
4.2	Functorial properties	72
4.3	The tensor algebra of a free module	74
4.4	Covariant extension of a tensor algebra	75
4.5	Derivations and skew derivations on a tensor algebra	76
4.6	Comments and exercises	78
4.7	Solutions to selected exercises	80
5	The exterior algebra of a module	84
	General remarks	84
5.1	The exterior algebra	84
5.2	Functorial properties	87
5.3	The exterior algebra of a free module	89
5.4	The exterior algebra of a direct sum	93
5.5	Covariant extension of an exterior algebra	95
5.6	Skew derivations on an exterior algebra	96
5.7	Pfaffians	100
5.8	Comments and exercises	105
5.9	Solutions to selected exercises	111
6	The symmetric algebra of a module	117
	General remarks	117
6.1	The symmetric algebra	118
6.2	Functorial properties	120
6.3	The symmetric algebra of a free module	121
6.4	The symmetric algebra of a direct sum	121
6.5	Covariant extension of a symmetric algebra	122
6.6	Derivations on a symmetric algebra	123
6.7	Differential operators	124
6.8	Comments and exercises	126
7	Coalgebras and Hopf algebras	130
	General remarks	130
7.1	A fresh look at algebras	130
7.2	Coalgebras	133
7.3	Graded coalgebras	134
7.4	Tensor products of coalgebras	135
7.5	Modified tensor products of coalgebras	143
7.6	Commutative and skew-commutative coalgebras	150
7.7	Linear forms on a coalgebra	151
7.8	Hopf algebras	153

7.9	Tensor products of Hopf algebras	156
7.10	$E(M)$ as a (modified) Hopf algebra	158
7.11	The Grassmann algebra of a module	160
7.12	$S(M)$ as a Hopf algebra	163
7.13	Comments and exercises	166
7.14	Solutions to selected exercises	169
8	Graded duality	175
	General remarks	175
8.1	Modules of linear forms	175
8.2	The graded dual of a graded module	178
8.3	Graded duals of algebras and coalgebras	184
8.4	Graded duals of Hopf algebras	188
8.5	Comments and exercises	191
8.6	Solutions to selected exercises	194
	Index	197