

# TABLE OF CONTENTS

PREFACES .....	vii
ABBREVIATIONS .....	xiii

## CHAPTER I

### INTRODUCTION

1. Some basic theorems .....	1
2. The zeros of the derivative .....	6
3. Physical interpretations .....	7
4. Geometric interpretation .....	9
5. Function-theoretic interpretations. Infrapolynomials .....	13

## CHAPTER II

### THE CRITICAL POINTS OF A POLYNOMIAL

6. The convex hull of critical points .....	21
7. The critical points of a real polynomial .....	25
8. Some generalizations .....	29
9. Polynomial solutions of Lamé's differential equation .....	36

## CHAPTER III

### INVARIANTIVE FORMULATION

10. The derivative under linear transformations .....	43
11. Covariant force fields .....	45
12. Circular regions .....	48
13. Zeros of the polar derivative .....	49
14. Generalization to abstract spaces .....	55

## CHAPTER IV

### COMPOSITE POLYNOMIALS

15. Apolar polynomials .....	60
16. Applications .....	65
17. Linear combinations of polynomials .....	74
18. Combinations of a polynomial and its derivatives .....	81

## CHAPTER V

### THE CRITICAL POINTS OF A RATIONAL FUNCTION WHICH HAS ITS ZEROS AND POLES IN PRESCRIBED CIRCULAR REGIONS

19. A two-circle theorem for polynomials .....	89
20. Two-circle theorems for rational functions .....	93
21. The general case .....	96
22. Some important special cases .....	102

## CHAPTER VI

THE CRITICAL POINTS OF A POLYNOMIAL WHICH  
HAS ONLY SOME PRESCRIBED ZEROS

23. Polynomials with two given zeros .....	107
24. Mean-Value Theorems .....	110
25. Polynomials with $p$ known zeros .....	113
26. Alternative treatment .....	118

## CHAPTER VII

## BOUNDS FOR THE ZEROS AS FUNCTIONS OF ALL THE COEFFICIENTS

27. The moduli of the zeros .....	122
28. The $p$ zeros of smallest modulus .....	128
29. Refinement of the bounds .....	130
30. Applications .....	133
31. Matrix methods .....	139

## CHAPTER VIII

BOUNDS FOR  $p$  ZEROS AS FUNCTIONS OF  $p + 1$  COEFFICIENTS

32. Construction of bounds .....	147
33. Further bounds .....	151
34. Lacunary polynomials .....	156
35. Other bounds for lacunary polynomials .....	163

## CHAPTER IX

## THE NUMBER OF ZEROS IN A HALF-PLANE OR A SECTOR

36. Dynamic stability .....	166
37. Cauchy indices .....	168
38. Sturm sequences .....	171
39. Determinant sequences .....	174
40. The number of zeros with negative real parts .....	179
41. The number of zeros in a sector .....	189

## CHAPTER X

## THE NUMBER OF ZEROS IN A GIVEN CIRCLE

42. An algorithm .....	194
43. Determinant sequences .....	198
44. Polynomials with zeros on or symmetric in the unit circle .....	201
45. Singular determinant sequences .....	203

BIBLIOGRAPHY .....	207
--------------------	-----

INDEX .....	241
-------------	-----