

# TABLE OF CONTENTS

## INTRODUCTION

### CHAPTER 1. POLYNOMIALS AND POLYNOMIAL FUNCTIONS

1. Basic concepts of universal algebra .....	1
2. Varieties .....	6
3. Free algebras, free unions, and free products .....	9
4. Polynomial algebras .....	12
5. The lattice of polynomial algebras over an algebra .....	16
6. Functions and polynomial functions on algebras .....	20
7. Normal forms of polynomials .....	22
8. Polynomials over commutative rings with identity .....	23
9. Polynomials over groups .....	27
10. Polynomials over lattices and Boolean algebras .....	31
11. Polynomially complete algebras .....	34
12. Some examples of polynomially complete algebras.....	38
Remarks and comments .....	41

### CHAPTER 2. ALGEBRAIC EQUATIONS

1. Systems of algebraic equations .....	47
2. Maximal systems of algebraic equations .....	52
3. Algebraically closed algebras .....	56
4. Algebraic independence .....	58
5. Systems of algebraic equations over groups. Algebraically closed groups	64
Remarks and comments .....	70

### CHAPTER 3. COMPOSITION OF POLYNOMIALS AND POLYNOMIAL FUNCTIONS

1. Composition algebras .....	73
2. Composition algebras of polynomials and polynomial functions .....	77
3. Composition homomorphisms .....	78
4. Full congruences .....	84
5. Full ideals over multioperator groups .....	89
6. Full ideals over commutative rings with identity.....	93
7. Full ideals over fields .....	97
8. Residue polynomial ideals of Dedekind domains .....	101

9. Residue polynomial ideals over groups .....	106
10. Derivation families with chain rule .....	108
11. Polynomial vectors and polynomial function vectors .....	112
12. Permutation polynomials and polynomial permutations .....	119
13. Subsemigroups defined by parametric words .....	123
Remarks and comments .....	130

## CHAPTER 4. COMPOSITION OF POLYNOMIALS AND POLYNOMIAL FUNCTIONS OVER RINGS AND FIELDS

1. Prime factor decomposition with respect to composition .....	134
2. Standard solutions of $p \circ q = r \circ s$ .....	138
3. Permutable chains over fields and integral domains .....	154
4. Permutation polynomial vectors and permutation polynomials over rings .....	161
5. Semigroups of polynomial function vectors and polynomial permutations over finite factor rings of Dedekind domains .....	167
6. Ideal power semigroups .....	180
7. Ideal power semigroups over factor rings of Dedekind domains .....	183
8. Characterization of permutation polynomials over finite fields .....	190
9. Semigroups of permutation polynomials and groups of polynomial permutations over finite fields .....	204
10. Permutation spectra of polynomials .....	213
Remarks and comments .....	219

## CHAPTER 5. COMPOSITION OF POLYNOMIALS AND POLYNOMIAL FUNCTIONS OVER GROUPS

1. The concept of length .....	223
2. Distributively generated composition groups and polynomial functions over groups .....	228
3. On polynomial permutations over groups .....	233
4. Further results on the group of polynomial permutations over a finite group .....	238
5. Characterization of classes of groups by properties of their permutation polynomials .....	244
Remarks and comments .....	248

## CHAPTER 6. APPENDIX

1. Sets .....	249
2. Lattices .....	255
3. Multioperator groups .....	257
4. Rings .....	260
5. Fields .....	269

6. Semigroups and groups .....	277
7. Linear algebra and representation theory .....	287
8. Near rings .....	291
9. Miscellaneous .....	296
Remarks and comments .....	299
 BIBLIOGRAPHY .....	 300
 AUTHOR INDEX .....	 313
 SUBJECT INDEX .....	 316