

CONTENTS

Preface	vii
Preface to the 2002 edition	xi
1 Notation and prerequisites	1
1.1 Algebra	1
1.2 Topology	5
1.3 Additional notes and comments	7a
2 Convex sets	8
2.1 Definition and elementary properties	8
2.2 Support and separation	10
2.3 Convex hulls	14
2.4 Extreme and exposed points; faces and poonems	17
2.5 Unbounded convex sets	23
2.6 Polyhedral sets	26
2.7 Remarks	28
2.8 Additional notes and comments	30a
3 Polytopes	31
3.1 Definition and fundamental properties	31
3.2 Combinatorial types of polytopes; complexes	38
3.3 Diagrams and Schlegel diagrams	42
3.4 Duality of polytopes	46
3.5 Remarks	51
3.6 Additional notes and comments	52a
4 Examples	53
4.1 The d -simplex	53
4.2 Pyramids	54
4.3 Bipyramids	55
4.4 Prisms	56
4.5 Simplicial and simple polytopes	57
4.6 Cubical polytopes	59
4.7 Cyclic polytopes	61
4.8 Exercises	63
4.9 Additional notes and comments	69a

5	Fundamental properties and constructions	70
5.1	Representations of polytopes as sections or projections	71
5.2	The inductive construction of polytopes	78
5.3	Lower semicontinuity of the functions $f_k(P)$	83
5.4	Gale-transforms and Gale-diagrams	85
5.5	Existence of combinatorial types	90
5.6	Additional notes and comments	96a
6	Polytopes with few vertices	97
6.1	d -Polytopes with $d + 2$ vertices	97
6.2	d -Polytopes with $d + 3$ vertices	102
6.3	Gale diagrams of polytopes with few vertices	108
6.4	Centrally symmetric polytopes	114
6.5	Exercises	119
6.6	Remarks	121
6.7	Additional notes and comments	121a
7	Neighborly polytopes	122
7.1	Definition and general properties	122
7.2	$[\frac{1}{2}d]$ -Neighborly d -polytopes	123
7.3	Exercises	125
7.4	Remarks	127
7.5	Additional notes and comments	129a
8	Euler's relation	130
8.1	Euler's theorem	130
8.2	Proof of Euler's theorem	134
8.3	A generalization of Euler's relation	137
8.4	The Euler characteristic of complexes	138
8.5	Exercises	139
8.6	Remarks	141
8.7	Additional notes and comments	142a
9	Analogues of Euler's relation	143
9.1	The incidence equation	143
9.2	The Dehn-Sommerville equations	145
9.3	Quasi-simplicial polytopes	153
9.4	Cubical polytopes	155
9.5	Solutions of the Dehn-Sommerville equations	160
9.6	The f -vectors of neighborly d -polytopes	162
9.7	Exercises	168

9.8	Remarks	170
9.9	Additional notes and comments	171a
10	Extremal problems concerning numbers of faces	172
10.1	Upper bounds for $f_i, i \geq 1$, in terms of f_0	172
10.2	Lower bounds for $f_i, i \geq 1$, in terms of f_0	183
10.3	The sets $f(\mathcal{P}^3)$ and $f(\mathcal{P}_3^3)$	189
10.4	The set $f(\mathcal{P}^4)$	191
10.5	Exercises	197
10.6	Additional notes and comments	198a
11	Properties of boundary complexes	199
11.1	Skeletons of simplices contained in $\mathcal{B}(P)$	200
11.2	A proof of the van Kampen-Flores theorem	210
11.3	d -Connectedness of the graphs of d -polytopes	212
11.4	Degree of total separability	217
11.5	d -Diagrams	218
11.6	Additional notes and comments	224a
12	k-Equivalence of polytopes	225
12.1	k -Equivalence and ambiguity	225
12.2	Dimensional ambiguity	226
12.3	Strong and weak ambiguity	228
12.4	Additional notes and comments	234a
13	3-Polytopes	235
13.1	Steinitz's theorem	235
13.2	Consequences and analogues of Steinitz's theorem	244
13.3	Eberhard's theorem	253
13.4	Additional results on 3-realizable sequences	271
13.5	3-Polytopes with circumspheres and circumcircles	284
13.6	Remarks	288
13.7	Additional notes and comments	296a
14	Angle-sums relations; the Steiner point	297
14.1	Gram's relation for angle-sums	297
14.2	Angle-sums relations for simplicial polytopes	304
14.3	The Steiner point of a polytope (by G. C. Shephard)	307
14.4	Remarks	312
14.5	Additional notes and comments	315a

15 Addition and decomposition of polytopes (by G. C. Shephard)	316
15.1 Vector addition	316
15.2 Approximation of polytopes by vector sums	324
15.3 Blaschke addition	331
15.4 Remarks	337
15.5 Additional notes and comments	340a
16 Diameters of polytopes (by Victor Klee)	341
16.1 Extremal diameters of d -polytopes	342
16.2 The functions Δ and Δ_b	347
16.3 W_v Paths	354
16.4 Additional notes and comments	355a
17 Long paths and circuits on polytopes (by Victor Klee)	356
17.1 Hamiltonian paths and circuits	357
17.2 Extremal path-lengths of polytopes	366
17.3 Heights of polytopes	375
17.4 Circuit codes	381
17.5 Additional notes and comments	389a
18 Arrangements of hyperplanes	390
18.1 d -Arrangements	390
18.2 2-Arrangements	397
18.3 Generalizations	407
18.4 Additional notes and comments	410a
19 Concluding remarks	411
19.1 Regular polytopes and related notions	411
19.2 k -Content of polytopes	416
19.3 Antipodality and related notions	418
19.4 Additional notes and comments	423a
Tables	424
Addendum	426
Errata for the 1967 edition	428a
Bibliography	429
Additional Bibliography	448a
Index of Terms	449
Index of Symbols	467