

Contents

<i>Preface</i>	XVII
<i>Introduction</i>	1

I

CLASSICAL ELECTRODYNAMICS: THE FUNDAMENTAL EQUATIONS AND THE DYNAMICAL VARIABLES

<i>Introduction</i>	5
A. The Fundamental Equations in Real Space	7
1. The Maxwell-Lorentz Equations	7
2. Some Important Constants of the Motion	8
3. Potentials—Gauge Invariance	8
B. Electrodynamics in Reciprocal Space	11
1. The Fourier Spatial Transformation—Notation	11
2. The Field Equations in Reciprocal Space	12
3. Longitudinal and Transverse Vector Fields	13
4. Longitudinal Electric and Magnetic Fields	15
5. Contribution of the Longitudinal Electric Field to the Total Energy, to the Total Momentum, and to the Total Angular Momentum— <i>a. The Total Energy. b. The Total Momentum. c. The Total Angular Momentum</i>	17
6. Equations of Motion for the Transverse Fields	21
C. Normal Variables	23
1. Introduction	23
2. Definition of the Normal Variables	23
3. Evolution of the Normal Variables	24
4. The Expressions for the Physical Observables of the Transverse Field as a Function of the Normal Variables— <i>a. The Energy H_{trans} of the Transverse Field. b. The Momentum $\mathbf{P}_{\text{trans}}$ and the Angular Momentum $\mathbf{J}_{\text{trans}}$ of the Transverse Field. c. Transverse Electric and Magnetic Fields in Real Space. d. The Transverse Vector Potential $\mathbf{A}_{\perp}(\mathbf{r}, t)$</i> . . .	26

5. Similarities and Differences between the Normal Variables and the Wave Function of a Spin-1 Particle in Reciprocal Space	30
6. Periodic Boundary Conditions. Simplified Notation	31
D. Conclusion: Discussion of Various Possible Quantization Schemes	33
1. Elementary Approach	33
2. Lagrangian and Hamiltonian Approach	34

COMPLEMENT A₁—THE “TRANSVERSE” DELTA FUNCTION

1. Definition in Reciprocal Space— <i>a. Cartesian Coordinates. Transverse and Longitudinal Components. b. Projection on the Subspace of Transverse Fields</i>	36
2. The Expression for the Transverse Delta Function in Real Space— <i>a. Regularization of $\delta_{ij}^{\perp}(\rho)$. b. Calculation of $g(\rho)$. c. Evaluation of the Derivatives of $g(\rho)$. d. Discussion of the Expression for $\delta_{ij}^{\perp}(\rho)$</i>	38
3. Application to the Evaluation of the Magnetic Field Created by a Magnetization Distribution. Contact Interaction	42

COMPLEMENT B₁—ANGULAR MOMENTUM OF THE ELECTROMAGNETIC FIELD. MULTIPOLE WAVES

<i>Introduction</i>	45
1. Contribution of the Longitudinal Electric Field to the Total Angular Momentum	45
2. Angular Momentum of the Transverse Field— <i>a. $\mathbf{J}_{\text{trans}}$ in Reciprocal Space. b. $\mathbf{J}_{\text{trans}}$ in Terms of Normal Variables. c. Analogy with the Mean Value of the Total Angular Momentum of a Spin-1 Particle</i>	47
3. Set of Vector Functions of \mathbf{k} “Adapted” to the Angular Momentum— <i>a. General Idea. b. Method for Constructing Vector Eigenfunctions for \mathbf{J}^2 and J_z. c. Longitudinal Eigenfunctions. d. Transverse Eigenfunctions</i>	51
4. Application: Multipole Waves in Real Space— <i>a. Evaluation of Some Fourier Transforms. b. Electric Multipole Waves. c. Magnetic Multipole Waves</i>	55

COMPLEMENT C₁—EXERCISES

1. H and \mathbf{P} as Constants of the Motion	61
2. Transformation from the Coulomb Gauge to the Lorentz Gauge	63
3. Cancellation of the Longitudinal Electric Field by the Instantaneous Transverse Field	64

4. Normal Variables and Retarded Potentials	66
5. Field Created by a Charged Particle at Its Own Position. Radiation Reaction	68
6. Field Produced by an Oscillating Electric Dipole	71
7. Cross-section for Scattering of Radiation by a Classical Elastically Bound Electron	74

II

**LAGRANGIAN AND HAMILTONIAN APPROACH
TO ELECTRODYNAMICS. THE STANDARD LAGRANGIAN
AND THE COULOMB GAUGE**

<i>Introduction</i>	79
A. Review of the Lagrangian and Hamiltonian Formalism	81
1. Systems Having a Finite Number of Degrees of Freedom— <i>a. Dynamical Variables, the Lagrangian, and the Action. b. Lagrange's Equations. c. Equivalent Lagrangians. d. Conjugate Momenta and the Hamiltonian. e. Change of Dynamical Variables. f. Use of Complex Generalized Coordinates. g. Coordinates, Momenta, and Hamiltonian in Quantum Mechanics.</i>	81
2. A System with a Continuous Ensemble of Degrees of Freedom— <i>a. Dynamical Variables. b. The Lagrangian. c. Lagrange's Equations d. Conjugate Momenta and the Hamiltonian. e. Quantization. f. Lagrangian Formalism with Complex Fields. g. Hamiltonian Formalism and Quantization with Complex Fields</i>	90
B. The Standard Lagrangian of Classical Electrodynamics	100
1. The Expression for the Standard Lagrangian— <i>a. The Standard Lagrangian in Real Space. b. The Standard Lagrangian in Reciprocal Space</i>	100
2. The Derivation of the Classical Electrodynamical Equations from the Standard Lagrangian— <i>a. Lagrange's Equation for Particles. b. The Lagrange Equation Relative to the Scalar Potential. c. The Lagrange Equation Relative to the Vector Potential</i>	103
3. General Properties of the Standard Lagrangian— <i>a. Global Symmetries. b. Gauge Invariance. c. Redundancy of the Dynamical Variables</i>	105
C. Electrodynamics in the Coulomb Gauge	111
1. Elimination of the Redundant Dynamical Variables from the Standard Lagrangian— <i>a. Elimination of the Scalar Potential. b. The Choice of the Longitudinal Component of the Vector Potential</i>	111
2. The Lagrangian in the Coulomb Gauge	113

3. Hamiltonian Formalism— <i>a. Conjugate Particle Momenta. b. Conjugate Momenta for the Field Variables. c. The Hamiltonian in the Coulomb Gauge. d. The Physical Variables</i>	115
4. Canonical Quantization in the Coulomb Gauge— <i>a. Fundamental Commutation Relations. b. The Importance of Transversability in the Case of the Electromagnetic Field. c. Creation and Annihilation Operators</i>	118
5. Conclusion: Some Important Characteristics of Electrodynamics in the Coulomb Gauge— <i>a. The Dynamical Variables Are Independent. b. The Electric Field Is Split into a Coulomb Field and a Transverse Field. c. The Formalism Is Not Manifestly Covariant. d. The Interaction of the Particles with Relativistic Modes Is Not Correctly Described</i> . .	121

COMPLEMENT A_{II}—FUNCTIONAL DERIVATIVE. INTRODUCTION AND A FEW APPLICATIONS

1. From a Discrete to a Continuous System. The Limit of Partial Derivatives	126
2. Functional Derivative	128
3. Functional Derivative of the Action and the Lagrange Equations	128
4. Functional Derivative of the Lagrangian for a Continuous System	130
5. Functional Derivative of the Hamiltonian for a Continuous System	132

COMPLEMENT B_{II}—SYMMETRIES OF THE LAGRANGIAN IN THE COULOMB GAUGE AND THE CONSTANTS OF THE MOTION

1. The Variation of the Action between Two Infinitesimally Close Real Motions	134
2. Constants of the Motion in a Simple Case	136
3. Conservation of Energy for the System Charges + Field	137
4. Conservation of the Total Momentum	138
5. Conservation of the Total Angular Momentum	139

COMPLEMENT C_{II}—ELECTRODYNAMICS IN THE PRESENCE OF AN EXTERNAL FIELD

1. Separation of the External Field	141
2. The Lagrangian in the Presence of an External Field— <i>a. Introduction of a Lagrangian. b. The Lagrangian in the Coulomb Gauge</i>	142
3. The Hamiltonian in the Presence of an External Field— <i>a. Conjugate Momenta. b. The Hamiltonian. c. Quantization</i>	143

COMPLEMENT D_{II}—EXERCISES

1. An Example of a Hamiltonian Different from the Energy	146
2. From a Discrete to a Continuous System: Introduction of the Lagrangian and Hamiltonian Densities	147
3. Lagrange's Equations for the Components of the Electromagnetic Field in Real Space	150
4. Lagrange's Equations for the Standard Lagrangian in the Coulomb Gauge	151
5. Momentum and Angular Momentum of an Arbitrary Field	152
6. A Lagrangian Using Complex Variables and Linear in Velocity	154
7. Lagrangian and Hamiltonian Descriptions of the Schrödinger Matter Field	157
8. Quantization of the Schrödinger Field	161
9. Schrödinger Equation of a Particle in an Electromagnetic Field: Arbitrariness of Phase and Gauge Invariance	167

III

QUANTUM ELECTRODYNAMICS IN THE
COULOMB GAUGE

<i>Introduction</i>	169
A. The General Framework	171
1. Fundamental Dynamical Variables. Commutation Relations	171
2. The Operators Associated with the Various Physical Variables of the System	171
3. State Space	175
B. Time Evolution	176
1. The Schrödinger Picture	176
2. The Heisenberg Picture. The Quantized Maxwell-Lorentz Equations— <i>a. The Heisenberg Equations for Particles. b. The Heisenberg Equations for Fields. c. The Advantages of the Heisenberg Point of View.</i>	176
C. Observables and States of the Quantized Free Field	183
1. Review of Various Observables of the Free Field— <i>a. Total Energy and Total Momentum of the Field. b. The Fields at a Given Point \mathbf{r} of Space. c. Observables Corresponding to Photoelectric Measurements</i>	183
2. Elementary Excitations of the Quantized Free Field. Photons— <i>a. Eigenstates of the Total Energy and the Total Momentum.</i>	

<i>b. The Interpretation in Terms of Photons. c. Single-Photon States. Propagation</i>	186
3. Some Properties of the Vacuum— <i>a. Qualitative Discussion. b. Mean Values and Variances of the Vacuum Field. c. Vacuum Fluctuations</i>	189
4. Quasi-classical States— <i>a. Introducing the Quasi-classical States. b. Characterization of the Quasi-classical States. c. Some Properties of the Quasi-classical States. d. The Translation Operator for a and a^+</i>	192
D. The Hamiltonian for the Interaction between Particles and Fields	197
1. Particle Hamiltonian, Radiation Field Hamiltonian, Interaction Hamiltonian	197
2. Orders of Magnitude of the Various Interactions Terms for Systems of Bound Particles	198
3. Selection Rules	199
4. Introduction of a Cutoff	200

COMPLEMENT A_{III}—THE ANALYSIS OF INTERFERENCE PHENOMENA IN THE QUANTUM THEORY OF RADIATION

<i>Introduction</i>	204
1. A Simple Model	205
2. Interference Phenomena Observable with Single Photodetection Signals— <i>a. The General Case. b. Quasi-classical States. c. Factored States. d. Single-Photon States</i>	206
3. Interference Phenomena Observable with Double Photodetection Signals— <i>a. Quasi-classical States. b. Single-Photon States. c. Two-Photon States</i>	209
4. Physical Interpretation in Terms of Interference between Transition Amplitudes	213
5. Conclusion: The Wave-Particle Duality in the Quantum Theory of Radiation	215

COMPLEMENT B_{III}—QUANTUM FIELD RADIATED BY CLASSICAL SOURCES

1. Assumptions about the Sources	217
2. Evolution of the Fields in the Heisenberg Picture	217
3. The Schrödinger Point of View. The Quantum State of the Field at Time t	219

COMPLEMENT C_{III}—COMMUTATION RELATIONS FOR FREE FIELDS AT DIFFERENT TIMES. SUSCEPTIBILITIES AND CORRELATION FUNCTIONS OF THE FIELDS IN THE VACUUM

<i>Introduction</i>	221
1. Preliminary Calculations	222
2. Field Commutators— <i>a. Reduction of the Expressions in Terms of D.</i> <i>b. Explicit Expressions for the Commutators.</i> <i>c. Properties of the Commutators</i>	223
3. Symmetric Correlation Functions of the Fields in the Vacuum	227

COMPLEMENT D_{III}—EXERCISES

1. Commutators of \mathbf{A} , \mathbf{E}_\perp , and \mathbf{B} in the Coulomb Gauge	230
2. Hamiltonian of a System of Two Particles with Opposite Charges Coupled to the Electromagnetic Field	232
3. Commutation Relations for the Total Momentum \mathbf{P} with H_P , H_R , and H_f	233
4. Bose–Einstein Distribution	234
5. Quasi-Probability Densities and Characteristic Functions	236
6. Quadrature Components of a Single-Mode Field. Graphical Representation of the State of the Field	241
7. Squeezed States of the Radiation Field	246
8. Generation of Squeezed States by Two-Photon Interactions	248
9. Quasi-Probability Density of a Squeezed State	250

IV
OTHER EQUIVALENT FORMULATIONS
OF ELECTRODYNAMICS

<i>Introduction</i>	253
A. How to Get Other Equivalent Formulations of Electrodynamics	255
1. Change of Gauge and of Lagrangian	255
2. Changes of Lagrangian and the Associated Unitary Transformation— <i>a. Changing the Lagrangian.</i> <i>b. The Two Quantum Descriptions.</i> <i>c. The Correspondence between the Two Quantum Descriptions.</i> <i>d. Application to the Electromagnetic Field</i>	256
3. The General Unitary Transformation. The Equivalence between the Different Formulations of Quantum Electrodynamics	262

B. Simple Examples Dealing with Charges Coupled to an External Field	266
1. The Lagrangian and Hamiltonian of the System	266
2. Simple Gauge Change; Gauge Invariance— <i>a. The New Description.</i> <i>b. The Unitary Transformation Relating the Two Descriptions—Gauge</i> <i>Invariance</i>	267
3. The Göppert–Mayer Transformation— <i>a. The Long-Wavelength Ap-</i> <i>proximation.</i> <i>b. Gauge Change Giving Rise to the Electric Dipole</i> <i>Interaction.</i> <i>c. The Advantages of the New Point of View.</i> <i>d. The</i> <i>Equivalence between the Interaction Hamiltonians $A \cdot p$ and $E \cdot r$.</i> <i>e. Generalizations</i>	269
4. A Transformation Which Does Not Reduce to a Change of Lagrangian: The Henneberger Transformation— <i>a. Motivation.</i> <i>b. De-</i> <i>termination of the Unitary Transformation.</i> <i>Transforms of the Various</i> <i>Operators.</i> <i>c. Physical Interpretation.</i> <i>d. Generalization to a Quan-</i> <i>tized Field: The Pauli–Fierz–Kramers Transformation</i>	275
C. The Power–Zienau–Woolley Transformation: The Multipole Form of the Interaction between Charges and Field	280
1. Description of the Sources in Terms of a Polarization and a Magneti- zation Density— <i>a. The Polarization Density Associated with a System</i> <i>of Charges.</i> <i>b. The Displacement.</i> <i>c. Polarization Current and Mag-</i> <i>netization Current</i>	280
2. Changing the Lagrangian— <i>a. The Power–Zienau–Woolley Transfor-</i> <i>mation.</i> <i>b. The New Lagrangian.</i> <i>c. Multipole Expansion of the In-</i> <i>teraction between the Charged Particles and the Field</i>	286
3. The New Conjugate Momenta and the New Hamiltonian— <i>a. The</i> <i>Expressions for These Quantities.</i> <i>b. The Physical Significance of the</i> <i>New Conjugate Momenta.</i> <i>c. The Structure of the New Hamiltonian</i>	289
4. Quantum Electrodynamics from the New Point of View— <i>a. Quanti-</i> <i>zation.</i> <i>b. The Expressions for the Various Physical Variables</i>	293
5. The Equivalence of the Two Points of View. A Few Traps to Avoid . . .	296
D. Simplified Form of Equivalence for the Scattering S-Matrix	298
1. Introduction of the S-Matrix	298
2. The S-Matrix from Another Point of View. An Examination of the Equivalence	300
3. Comments on the Use of the Equivalence between the S-Matrices . . .	302

COMPLEMENT A_{IV}—ELEMENTARY INTRODUCTION
TO THE ELECTRIC DIPOLE HAMILTONIAN

<i>Introduction</i>	304
1. The Electric Dipole Hamiltonian for a Localized System of Charges Coupled to an External Field— <i>a. The Unitary Transformation Suggested</i>	

by the Long-Wavelength Approximation. b. The Transformed Hamiltonian. c. The Velocity Operator in the New Representation 304

2. The Electric Dipole Hamiltonian for a Localized System of Charges Coupled to Quantized Radiation—*a. The Unitary Transformation. b. Transformation of the Physical Variables. c. Polarization Density and Displacement. d. The Hamiltonian in the New Representation* 307

3. Extensions—*a. The Case of Two Separated Systems of Charges. b. The Case of a Quantized Field Coupled to Classical Sources* 312

COMPLEMENT B_{IV}—ONE-PHOTON AND TWO-PHOTON PROCESSES:
THE EQUIVALENCE BETWEEN THE INTERACTION
HAMILTONIANS $\mathbf{A} \cdot \mathbf{p}$ AND $\mathbf{E} \cdot \mathbf{r}$

Introduction 316

1. Notations. Principles of Calculations 316

2. Calculation of the Transition Amplitudes in the Two Representations—*a. The Interaction Hamiltonian $\mathbf{A} \cdot \mathbf{p}$. b. The Interaction Hamiltonian $\mathbf{E} \cdot \mathbf{r}$. c. Direct Verification of the Identity of the Two Amplitudes* 317

3. Generalizations—*a. Extension to Other Processes. b. Nonresonant Processes* 325

COMPLEMENT C_{IV}—INTERACTION OF TWO LOCALIZED SYSTEMS
OF CHARGES FROM THE POWER-ZIENAU-WOOLLEY
POINT OF VIEW

Introduction 328

1. Notation 328

2. The Hamiltonian 329

COMPLEMENT D_{IV}—THE POWER-ZIENAU-WOOLLEY
TRANSFORMATION AND THE POINCARÉ GAUGE

Introduction 331

1. The Power-Zienau-Woolley Transformation Considered as a Gauge Change 331

2. Properties of the Vector Potential in the New Gauge 332

3. The Potentials in the Poincaré Gauge 333

COMPLEMENT E_{IV}—EXERCISES

1. An Example of the Effect Produced by Sudden Variations of the Vector Potential	336
2. Two-Photon Excitation of the Hydrogen Atom. Approximate Results Obtained with the Hamiltonians $\mathbf{A} \cdot \mathbf{p}$ and $\mathbf{E} \cdot \mathbf{r}$	338
3. The Electric Dipole Hamiltonian for an Ion Coupled to an External Field	342
4. Scattering of a Particle by a Potential in the Presence of Laser Radiation	344
5. The Equivalence between the Interaction Hamiltonians $\mathbf{A} \cdot \mathbf{p}$ and $\mathbf{Z} \cdot (\nabla V)$ for the Calculation of Transition Amplitudes	349
6. Linear Response and Susceptibility. Application to the Calculation of the Radiation from a Dipole	352
7. Nonresonant Scattering. Direct Verification of the Equality of the Transition Amplitudes Calculated from the Hamiltonians $\mathbf{A} \cdot \mathbf{p}$ and $\mathbf{E} \cdot \mathbf{r}$	356

V

INTRODUCTION TO THE COVARIANT FORMULATION
OF QUANTUM ELECTRODYNAMICS

<i>Introduction</i>	361
A. Classical Electrodynamics in the Lorentz Gauge	364
1. Lagrangian Formalism— <i>a. Covariant Notation. Ordinary Notation. b. Selection of a New Lagrangian for the Field. c. Lagrange Equations for the Field. d. The Subsidiary Condition. e. The Lagrangian Density in Reciprocal Space</i>	364
2. Hamiltonian Formalism— <i>a. Conjugate Momenta of the Potentials. b. The Hamiltonian of the Field. c. Hamilton–Jacobi Equations for the Free Field</i>	369
3. Normal Variables of the Classical Field— <i>a. Definition. b. Expansion of the Potential in Normal Variables. c. Form of the Subsidiary Condition for the Free Classical Field. Gauge Arbitrariness. d. Expression of the Field Hamiltonian</i>	371
B. Difficulties Raised by the Quantization of the Free Field	380
1. Canonical Quantization — <i>a. Canonical Commutation Relations. b. Annihilation and Creation Operators. c. Covariant Commutation Relations between the Free Potentials in the Heisenberg Picture</i>	380
2. Problems of Physical Interpretation Raised by Covariant Quantization — <i>a. The Form of the Subsidiary Condition in Quantum Theory. b. Problems Raised by the Construction of State Space</i>	383

C. Covariant Quantization with an Indefinite Metric	387
1. Indefinite Metric in Hilbert Space	387
2. Choice of the New Metric for Covariant Quantization	390
3. Construction of the Physical Kets	393
4. Mean Values of the Physical Variables in a Physical Ket— <i>a. Mean Values of the Potentials and the Fields. b. Gauge Arbitrariness and Arbitrariness of the Kets Associated with a Physical State. c. Mean Value of the Hamiltonian</i>	396
D. A Simple Example of Interaction: A Quantized Field Coupled to Two Fixed External Charges	400
1. Hamiltonian for the Problem	400
2. Energy Shift of the Ground State of the Field. Reinterpretation of Coulomb's Law— <i>a. Perturbative Calculation of the Energy Shift. b. Physical Discussion. Exchange of Scalar Photons between the Two Charges. c. Exact Calculation</i>	401
3. Some Properties of the New Ground State of the Field— <i>a. The Subsidiary Condition in the Presence of the Interaction. The Physical Character of the New Ground State. b. The Mean Value of the Scalar Potential in the New Ground State of the Field</i>	405
4. Conclusion and Generalization	407

COMPLEMENT A_v—AN ELEMENTARY INTRODUCTION TO THE THEORY OF THE ELECTRON-POSITRON FIELD COUPLED TO THE PHOTON FIELD IN THE LORENTZ GAUGE

<i>Introduction</i>	408
1. A Brief Review of the Dirac Equation— <i>a. Dirac Matrices. b. The Dirac Hamiltonian. Charge and Current Density. c. Connection with the Covariant Notation. d. Energy Spectrum of the Free Particle. e. Negative-Energy States. Hole Theory</i>	408
2. Quantization of the Dirac Field— <i>a. Second Quantization. b. The Hamiltonian of the Quantized Field. Energy Levels. c. Temporal and Spatial Translations</i>	414
3. The Interacting Dirac and Maxwell Fields— <i>a. The Hamiltonian of the Total System. The Interaction Hamiltonian. b. Heisenberg Equations for the Fields. c. The Form of the Subsidiary Condition in the Presence of Interaction</i>	418

COMPLEMENT B_V —JUSTIFICATION OF THE NONRELATIVISTIC THEORY
IN THE COULOMB GAUGE STARTING FROM RELATIVISTIC
QUANTUM ELECTRODYNAMICS

<i>Introduction</i>	424
1. Transition from the Lorentz Gauge to the Coulomb Gauge in Relativistic Quantum Electrodynamics— <i>a. Transformation on the Scalar Photons Yielding the Coulomb Interaction. b. Effect of the Transformation on the Other Terms of the Hamiltonian in the Lorentz Gauge. c. Subsidiary Condition. Absence of Physical Effects of the Scalar and Longitudinal Photons. d. Conclusion: The Relativistic Quantum Electrodynamics Hamiltonian in the Coulomb Gauge</i>	425
2. The Nonrelativistic Limit in Coulomb Gauge: Justification of the Pauli Hamiltonian for the Particles— <i>a. The Dominant Term H_0 of the Hamiltonian in the Nonrelativistic Limit: Rest Mass Energy of the Particles. b. The Effective Hamiltonian inside a Manifold. c. Discussion</i>	432

COMPLEMENT C_V —EXERCISES

1. Other Covariant Lagrangians of the Electromagnetic Field	441
2. Annihilation and Creation Operators for Scalar Photons: Can One Interchange Their Meanings?	443
3. Some Properties of the Indefinite Metric	445
4. Translation Operator for the Creation and Annihilation Operators of a Scalar Photon	446
5. Lagrangian of the Dirac Field. The Connection between the Phase of the Dirac Field and the Gauge of the Electromagnetic Field	449
6. The Lagrangian and Hamiltonian of the Coupled Dirac and Maxwell Fields	451
7. Dirac Field Operators and Charge Density. A Study of Some Commutation Relations	454
<i>References</i>	457
<i>Index</i>	459