

CONTENTS

PART I. GENERAL CONCEPTS AND ONE-DIMENSIONAL GEOMETRY

CHAPTER I. GENERAL CONCEPTS

SECTION	PAGE
1. Coördinates	1
2. The principle of duality	2
3. The use of imaginaries	2
4. Infinity	3
5. Transformations	4
6. Groups	6

CHAPTER II. RANGES AND PENCILS

7. Cartesian coördinate of a point on a line	8
8. Projective coördinate of a point on a line	8
9. Change of coördinates	9
10. Coördinate of a line of a pencil	11
11. Coördinate of a plane of a pencil	12

CHAPTER III. PROJECTIVITY

12. The linear transformation	13
13. The cross ratio	16
14. Harmonic sets	18
15. Projection	20
16. Perspective figures	21
17. Other one-dimensional extents	23

PART II. TWO-DIMENSIONAL GEOMETRY

CHAPTER IV. POINT AND LINE COÖRDINATES IN A PLANE

18. Homogeneous Cartesian point coördinates	27
19. The straight line	27
20. The circle points at infinity	30
21. The conic	32
22. Trilinear point coördinates	34
23. Points on a line	35

SECTION	PAGE
24. The linear equation in point coördinates	36
25. Lines of a pencil	37
26. Line coördinates in a plane	38
27. Pencil of lines and the linear equation in line coördinates	39
28. Dualistic relations	40
29. Change of coördinates	41
30. Certain straight-line configurations	44
31. Curves in point coördinates	50
32. Curves in line coördinates	53

CHAPTER V. CURVES OF SECOND ORDER AND SECOND CLASS

33. Singular points of a curve of second order	58
34. Poles and polars with respect to a curve of second order	59
35. Classification of curves of second order	65
36. Singular lines of a curve of second class	67
37. Classification of curves of second class	68
38. Poles and polars with respect to a curve of second class	70
39. Projective properties of conics	72

CHAPTER VI. LINEAR TRANSFORMATIONS

40. Collineations	78
41. Types of nonsingular collineations	83
42. Correlations	88
43. Pairs of conics	95
44. The projective group	100
45. The metrical group	101
46. Angle and the circle points at infinity	105

CHAPTER VII. PROJECTIVE MEASUREMENT

47. General principles	107
48. The hyperbolic case	110
49. The elliptic case	115
50. The parabolic case	117

CHAPTER VIII. CONTACT TRANSFORMATIONS IN THE PLANE

51. Point-point transformations	120
52. Quadric inversion	121
53. Inversion	124
54. Point-curve transformations	127
55. The pedal transformation	131
56. The line element	133

CHAPTER IX. TETRACYCLICAL COÖRDINATES

SECTION	PAGE
57. Special tetracyclical coördinates	138
58. Distance between two points	139
59. The circle	140
60. Relation between tetracyclical and Cartesian coördinates	142
61. Orthogonal circles	144
62. Pencils of circles	146
63. The general tetracyclical coördinates	150
64. Orthogonal coördinates	153
65. The linear transformation	154
66. The metrical transformation	155
67. Inversion	156
68. The linear group	159
69. Duals of tetracyclical coördinates	161

CHAPTER X. A SPECIAL SYSTEM OF COÖRDINATES

70. The coördinate system	164
71. The straight line and the equilateral hyperbola	166
72. The bilinear equation	167
73. The bilinear transformation	169

PART III. THREE-DIMENSIONAL GEOMETRY

CHAPTER XI. CIRCLE COÖRDINATES

74. Elementary circle coördinates	171
75. The quadratic circle complex	173
76. Higher circle coördinates	177

CHAPTER XII. POINT AND PLANE COÖRDINATES

77. Cartesian point coördinates	180
78. Distance	181
79. The straight line	182
80. The plane	185
81. Direction and angle	188
82. Quadriplanar point coördinates	193
83. Straight line and plane	194
84. Plane coördinates	197
85. One-dimensional extents of points	200
86. Locus of an equation in point coördinates	205
87. One-dimensional extents of planes	210
88. Locus of an equation in plane coördinates	215
89. Change of coördinates	218

CHAPTER XIII. SURFACES OF SECOND ORDER AND OF
SECOND CLASS

SECTION	PAGE
90. Surfaces of second order	220
91. Singular points	221
92. Poles and polars	222
93. Classification of surfaces of second order	224
94. Surfaces of second order in Cartesian coördinates	227
95. Surfaces of second order referred to rectangular axes	229
96. Rulings on surfaces of second order	232
97. Surfaces of second class	235
98. Poles and polars	238
99. Classification of surfaces of the second class	238

CHAPTER XIV. TRANSFORMATIONS

100. Collineations	240
101. Types of nonsingular collineations	241
102. Correlations	246
103. The projective and the metrical groups	249
104. Projective geometry on a quadric surface	250
105. Projective measurement	253
106. Clifford parallels	255
107. Contact transformations	258
108. Point-point transformations	260
109. Point-surface transformations	262
110. Point-curve transformations	263

CHAPTER XV. THE SPHERE IN CARTESIAN COÖRDINATES

111. Pencils of spheres	266
112. Bundles of spheres	268
113. Complexes of spheres	269
114. Inversion	270
115. Dupin's cyclide	274
116. Cyclides	279

CHAPTER XVI. PENTASPHERICAL COÖRDINATES

117. Specialized coördinates	282
118. The sphere	284
119. Angle between spheres	286
120. The power of a point with respect to a sphere	287
121. General orthogonal coördinates	288

SECTION	PAGE
122. The linear transformation	291
123. Relation between pentaspherical and Cartesian coördinates . . .	293
124. Pencils, bundles, and complexes of spheres	293
125. Tangent circles and spheres	295
126. Cyclides in pentaspherical coördinates	297

PART IV. GEOMETRY OF FOUR AND HIGHER
DIMENSIONS

CHAPTER XVII. LINE COÖRDINATES IN THREE-
DIMENSIONAL SPACE

127. The Plücker coördinates	301
128. Dualistic definition	303
129. Intersecting lines	304
130. General line coördinates	305
131. Pencils and bundles of lines	306
132. Complexes, congruences, series	308
133. The linear line complex	310
134. Conjugate lines	314
135. Complexes in point coördinates	316
136. Complexes in Cartesian coördinates	317
137. The bilinear equation in point coördinates	321
138. The linear line congruence	322
139. The cylindroid	323
140. The linear line series	324
141. The quadratic line complex	328
142. Singular surface of the quadratic complex	331
143. Plücker's complex surfaces	334
144. The (2, 2) congruence	335
145. Line congruences in general	336

CHAPTER XVIII. SPHERE COÖRDINATES

146. Elementary sphere coördinates	341
147. Higher sphere coördinates	343
148. Angle between spheres	344
149. The linear complex of oriented spheres	346
150. Linear congruence of oriented spheres	348
151. Linear series of oriented spheres	349
152. Pencils and bundles of tangent spheres	350
153. Quadratic complex of oriented spheres	353
154. Duality of line and sphere geometry	357

CHAPTER XIX. FOUR-DIMENSIONAL POINT COÖRDINATES

SECTION	PAGE
155. Definitions	362
156. Intersections	365
157. Euclidean space of four dimensions	368
158. Parallelism	370
159. Perpendicularity	373
160. Minimum lines, planes, and hyperplanes	378
161. Hypersurfaces of second order	382
162. Duality between line geometry in three dimensions and point geometry in four dimensions	384

CHAPTER XX. GEOMETRY OF N DIMENSIONS

163. Projective space	388
164. Intersection of linear spaces	390
165. The quadratic hypersurface	392
166. Intersection of a quadric by hyperplanes	396
167. Linear spaces on a quadric	401
168. Stereographic projection of a quadric in S_n upon S'_{n-1}	407
169. Application to line geometry	410
170. Metrical space of n dimensions	413
171. Minimum projection of S_n upon S_{n-1}	419
INDEX	421