

Contents

1. Measure and Category on the Line	1
Countable sets, sets of first category, nullsets, the theorems of Cantor, Baire, and Borel	
2. Liouville Numbers	6
Algebraic and transcendental numbers, measure and category of the set of Liouville numbers	
3. Lebesgue Measure in r -Space.	10
Definitions and principal properties, measurable sets, the Lebesgue density theorem	
4. The Property of Baire	19
Its analogy to measurability, properties of regular open sets	
5. Non-Measurable Sets	22
Vitali sets, Bernstein sets, Ulam's theorem, inaccessible cardinals, the continuum hypothesis	
6. The Banach-Mazur Game	27
Winning strategies, category and local category, indeterminate games	
7. Functions of First Class	31
Oscillation, the limit of a sequence of continuous functions, Riemann integrability	
8. The Theorems of Lusin and Egoroff	36
Continuity of measurable functions and of functions having the property of Baire, uniform convergence on subsets	
9. Metric and Topological Spaces	39
Definitions, complete and topologically complete spaces, the Baire category theorem	
10. Examples of Metric Spaces.	42
Uniform and integral metrics in the space of continuous functions, integrable functions, pseudo-metric spaces, the space of measurable sets	
11. Nowhere Differentiable Functions	45
Banach's application of the category method	

12.	The Theorem of Alexandroff	47
	Remetrization of a G_δ subset, topologically complete subspaces	
13.	Transforming Linear Sets into Nullsets	49
	The space of automorphisms of an interval, effect of monotone substitution on Riemann integrability, nullsets equivalent to sets of first category	
14.	Fubini's Theorem	52
	Measurability and measure of sections of plane measurable sets	
15.	The Kuratowski-Ulam Theorem	56
	Sections of plane sets having the property of Baire, product sets, reducibility to Fubini's theorem by means of a product transformation	
16.	The Banach Category Theorem	62
	Open sets of first category or measure zero, Montgomery's lemma, the theorems of Marczewski and Sikorski, cardinals of measure zero, decomposition into a nullset and a set of first category	
17.	The Poincaré Recurrence Theorem	65
	Measure and category of the set of points recurrent under a nondissipative transformation, application to dynamical systems	
18.	Transitive Transformations	70
	Existence of transitive automorphisms of the square, the category method	
19.	The Sierpinski-Erdős Duality Theorem	74
	Similarities between the classes of sets of measure zero and of first category, the principle of duality	
20.	Examples of Duality	78
	Properties of Lusin sets and their duals, sets almost invariant under transformations that preserve nullsets or category	
21.	The Extended Principle of Duality	82
	A counter example, product measures and product spaces, the zero-one law and its category analogue	
22.	Category Measure Spaces	86
	Spaces in which measure and category agree, topologies generated by lower densities, the Lebesgue density topology	
	References	92
	Index	94