

Table of Contents

Preface	v
Introduction	1
I. Review of Basic Algebra	5
1. Introduction	5
2. Skewfields and Fields	5
3. Group Theory	9
4. Vector Spaces	13
II. Classical Projective Planes	17
1. Introduction	17
2. Basic Definitions and Results	19
3. Dual Spaces and Homogeneous Coordinates	22
4. Isomorphisms, the Fundamental Theorem and Related Topics	28
5. The Line in the Plane	36
6. Polarities	45
7. Conics	50
8. Units	57
9. Affine Planes	63
10. Singer Groups	71
11. Appendix (Characteristic Two)	74
III. Elementary Properties of Projective and Affine Planes	76
1. Introduction	76
2. Basic Concepts	77
3. Subplanes	81
4. Affine Planes	82
5. Incidence Matrices	85
6. Appendix (The Bruck-Ryser Theorem)	87
IV. Collineations	90
1. Introduction	90
2. Basic Concepts and Definitions	90

3. Quasiperspectivities	91
4. Perspectivities	94
5. (V, l) -Transitivity	99
6. Collineation Groups Containing Perspectivities	101
7. Desargues' Configuration	105
V. Coordinatization and Planar Ternary Rings	110
1. Introduction	110
2. Introduction of Coordinates	110
3. Planar Ternary Rings	113
4. Algebraic Properties of Planar Ternary Rings	116
5. Latin Squares	116
6. Appendix (Other Methods of Coordinatization)	121
VI. Algebraic Properties of Planar Ternary Rings	127
1. Introduction	127
2. The Condition for Linearity	127
3. Additive Properties of (R, T)	129
4. Multiplicative Properties of (R, T)	132
5. Coordinatizing the Dual Plane	134
6. Division Rings with Inverse Properties	135
7. Skornyakov-San Soucie Theorem	140
8. Artin-Zorn Theorem	149
9. Summary of Results from Previous Sections	153
10. Homomorphisms	155
VII. Quasifields and Translation Planes	157
1. Introduction	157
2. Algebraic Definitions and Basic Properties	157
3. Congruence Partitions	160
4. Properties of the Kernel	164
5. Spreads	166
VIII. Division Rings and Their Planes	169
1. Introduction	169
2. The Semi-nuclei	169
3. Collineation Groups of Division Ring Planes	172
4. Autotopisms	174
5. Isotopism	176
6. On the Solubility of the Autotopism Group	178

IX. Some Examples of Non-desarguesian Planes	183
1. Introduction	183
2. The Hall Quasifields	183
3. The André Quasifields	178
4. A Class of Division Rings	191
5. The Dickson Commutative Division Rings	195
6. The Hughes Planes	196
 X. Derivation	202
1. Introduction	202
2. The Basic Results and Definitions	202
3. Desarguesian Planes	205
4. General Derivation	210
5. Inherited Collineation Groups	216
6. Partial Spreads	217
 XI. Free Closures	219
1. Introduction	219
2. Configurations	219
3. Generation	221
4. Classification Problems	227
5. Collineations	233
 XII. Polarities	238
1. Introduction	238
2. Absolute Elements of Polarities	239
3. Ovals	242
4. Polarities of Planes of Square Order	244
5. Polarities in Translation Planes	247
6. Appendix (Segre's Theorem)	250
 XIII. Collineation Groups	253
1. Introduction	253
2. Some Orbit Theorems	253
3. Collineation Groups Containing Perspectivities	257
4. Some Characterizations of Desarguesian Planes	260
5. Singer Groups and Difference Sets	262
6. Some Non-existence Theorems	267
7. Appendix (Multiplier Theorem)	269

XIV. Wagner's Theorem	272
1. Introduction	272
2. Some Preliminary Results on Permutation Groups	272
3. Two Preliminary Theorems	273
4. Some Special Cases	274
5. Planes of Type \mathcal{H}	276
6. Two-subplanes	277
7. Proof of Wagner's Theorem	277
8. Appendix (Proof of Some Permutation Group Theorems)	281
XV. Appendix	284
Index	289