

PART I**SUPERSINGULAR AND FIXED TRACE DISTRIBUTION****PRELIMINARIES**

1. The Galois representation in GL_2	17
2. Some notions of probability	20

THE DISTRIBUTION FOR FIXED TRACE

3. The probabilistic model	29
4. The asymptotic behavior	33

EXAMPLES

5. Serre curves, $M = 2q$, the general formula	41
6. Computations of Galois groups	46
7. The curve $y^2 = x^3 + 6x - 2$	51
8. The Shimura curve $X_0(11)$	55

PART II**IMAGINARY QUADRATIC DISTRIBUTION**

Introduction	69
---------------------	-----------

THE FIXED TRACE CASE

1. Fixed traces from the quadratic field	77
2. Computation of the constant for fixed trace	84

THE MODEL FOR THE MIXED CASE

3. The mixed Galois representations	91
4. The probabilistic model	104
5. The asymptotic behavior	108
6. The finite part of the constant as a quotient of integrals	112

COMPUTATIONS OF HARISH TRANSFORMS

7. Haar measure under the trace-determinant map on Mat_2 . General formalism.	123
8. Relations with the trace-norm map on k	133
9. Computation of C_ℓ for almost all ℓ	141
10. The constant for Serre curves, $K \cap k_{ab} = Q_{ab}$	143
11. The constant for $X_0(11)$	149

PART III

SPECIAL COMPUTATIONS

Introduction

157

GENERAL LEMMAS

1.	Lemmas on commutator subgroups	163
2.	$G_2 = \mathrm{GL}_2(\mathbb{Z}_2)$	165
3.	Cases when $K \cap k_{ab} = \mathbb{Q}_{ab}$	174
4.	$K \cap k_{ab}$ when $k = \mathbb{Q}(\sqrt{-3})$ and $\mathrm{GL}_2(\mathbb{Z}_3)$ splits	181
5.	$K \cap k_{ab}$ in other cases	185

$$k = \mathbb{Q}(\sqrt{-3})$$

6.	The action of \mathfrak{A} on $k(\Delta^{1/3})$	191
7.	The constant for Serre fiberings, $k = \mathbb{Q}(\sqrt{-3})$, $M = 2q$, q odd prime $\neq 3$, $\Delta = \pm q^n$	195
8.	Computation of integrals	201

$$k = \mathbb{Q}(i)$$

9.	The constant for Serre fiberings, q odd $\neq 3$	209
----	--	-----

$$k = \mathbb{Q}(\sqrt{\Delta})$$

10.	The action of \mathfrak{A} on $k(A_2, \Delta^{1/4})$ when $k = \mathbb{Q}(\sqrt{\Delta})$	215
11.	The action of matrices on $k(A_4)$	218
12.	Computation of integrals and the constant	221

PART IV

NUMERICAL RESULTS

SUPERSINGULAR AND FIXED TRACE DISTRIBUTION

1. General discussion of results	235
2. Tables	
Table I : Fixed trace distributions	239
Table II : Supersingular primes	240
Table III: Primes with $t_p = 1$	241
Table IV: Traces of Frobenius	242

IMAGINARY QUADRATIC DISTRIBUTION

3. General discussion of results	249
4. Tables	
Table V : Imaginary quadratic distributions	253
Table VI : Primes associated with fields of small discriminant, for curves A and B	258
Table VII: Distribution of primes associated with small discriminants	260

EXTENDED RESULTS FOR $X_0(11)$

5. Discussion and description of tables	265
Table VIII: Supersingular primes	267
Table IX : Imaginary quadratic distribution	268
Table X : Distribution of primes for fields with small discriminants	269

Remarks on the Computations	271
-----------------------------	-----

Bibliography	273
--------------	-----