

CONTENTS

I.	LOCALLY COMPACT GROUPS AND FIELDS	
1.	Haar measure	
1.1	Existence and uniqueness	1
1.2	Module of an automorphism	2
1.3	Homogeneous spaces	3
2.	Local and global fields	
2.1	Classification theorem	4
2.2	Structure of local fields	5
	Appendix: Review of number fields and completions . .	6
3.	Adele ring of a global field	
3.1	Restricted topological products	9
3.2	Adeles	10
II.	THE ADDITIVE GROUP	
4.	The quotient \mathbb{A}_K/K	
4.1	The space K_∞	11
4.2	Fundamental domain for K in \mathbb{A}_K	13
4.3	Product formula	14
5.	Volume of fundamental domain	
5.1	Normalized Haar measure	15
5.2	Volume calculation	16
5.3	Application: Fields of discriminant ± 1	16
6.	Strong approximation	
6.1	Chinese Remainder Theorem	18
6.2	An important lemma	19
6.3	Main theorem	20
III.	THE MULTIPLICATIVE GROUP	
7.	Ideles	
7.1	Idele topology	21
7.2	Special ideles	22
8.	Compactness theorem	
8.1	Compactness of J_K^0/K^*	24
8.2	Applications: Class number and units of K . .	25
8.3	Fundamental domain	28
IV.	GL_n AND SL_n (OVER R)	
9.	Example: The modular group	29
10.	Siegel sets in $GL(n, R)$	
10.1	Iwasawa decomposition	31
10.2	Siegel sets	33
10.3	Minimum principle	34

11.	Applications	
11.1	Siegel sets in $SL(n, R)$	36
11.2	Reduction of positive definite quadratic forms	39
12.	BN-pairs	
12.1	Axioms and Bruhat decomposition	41
12.2	Parabolic subgroups	43
12.3	Conjugates of B by W	46
12.4	Complements for GL_n	51
13.	Siegel property (and applications)	
13.1	Siegel sets revisited	53
13.2	Fundamental sets and Siegel property	56
13.3	Proof of Harish-Chandra's theorem	58
13.4	Finite presentation of Γ	61
13.5	Corners and arithmetic groups	64
V.	GL_n AND SL_n (p-ADIC AND ADELIC GROUPS)	
14.	Adelic groups	
14.1	Adelization of a linear group	65
14.2	Class number	66
14.3	Strong approximation	68
14.4	Reduction theory	70
15.	SL_2 (over p-adic fields)	
15.1	Infinite dihedral group	71
15.2	Lattices in K^2	72
15.3	BN-pair in G	73
15.4	Building attached to BN-pair	77
15.5	Ihara's theorem; maximal compact subgroups	78
	Appendix: Graphs and free groups	81
VI.	THE CONGRUENCE SUBGROUP PROBLEM	
16.	Reformulation of the problem	
16.1	Topological groups	85
16.2	Subgroup topologies on $SL(n, \mathbb{Q})$ and $SL(n, \mathbb{Z})$	86
16.3	Review of topology	88
16.4	Profinite groups	90
16.5	Completions of topological groups	92
16.6	The congruence kernel	97
17.	The congruence kernel of $SL(n, \mathbb{Z})$	
17.1	Some consequences of the invariant factor theorem	98
17.2	Congruence subgroups and q-elementary subgroups	100
17.3	A finiteness lemma	103
17.4	Proof of the theorem	104

17.5	The congruence kernel	106
17.6	Universal property of the extension	108
18.	The Steinberg group	
18.1	Generators and relations	109
18.2	The upper unitriangular group	110
18.3	The monomial group	111
18.4	Steinberg symbols	112
18.5	Determination of A	114
18.6	Determination of Ker ϕ	115
18.7	Universal property	116
18.8	Properties of Steinberg symbols	119
19.	Matsumoto's theorem	
19.1	Central extensions and cocycles	121
19.2	Statement of the theorem	123
19.3	The diagonal group	124
19.4	An auxiliary construction	124
19.5	The monomial group	129
19.6	Conclusion of the proof	132
19.7	The big cell	136
19.8	The topological case	137
20.	Moore's theory	
20.1	Topological Steinberg symbols	140
20.2	Local and global theorems	142
20.3	Central extensions of locally compact groups .	143
20.4	The fundamental group in the local case	144
20.5	Restricted products	145
20.6	Relative coverings	146
20.7	The congruence kernel revisited	148
SUGGESTIONS FOR FURTHER READING		149
REFERENCES		151
INDEX		157