CONTENTS

	Preface	iii
Chapter 1:	Orthogonal Designs--the problem and remarks on its solution	1
§1.1.	Generalities on Hurwitz-Radon families to prove there exist orthogonal designs of type $(1,1,\ldots,1)$	2
Chapter 2:	Some Algebraic and Combinatorial Nonexistence Results	7
§2.1.	Odd order (nonexistence)	8
§2.2.	$n = 2b$, b odd (nonexistence combinatorial and algebraic)	11
§2.3.	Statement of the algebraic problem for orthogonal designs	15
§2.4.	The Geramita-Verner Theorem and some consequences	17
Chapter 3:	The Algebraic Theory of Orthogonal Designs	21
§3.1.	Generalities on quadratic and bilinear forms	22
§3.2.	The matrix formulation	23
§3.3.	Mappings between bilinear spaces	25
§3.4.	New spaces from old	26
§3.5.	Classification theorems for bilinear spaces	27
§3.6.	Classification of quadratic forms over $\mathbb{Q}$	29
§3.7.	The similarities of a bilinear space	34
§3.8.	Linear subspaces of Sim(V)	35
§3.9.	Relations between rational families in the same order	42
§3.10.	Clifford algebras	43
§3.11.	Similarity representations	45
§3.12.	Some facts about positive definite forms over $\mathbb{Q}$	48

§3.13. Reduction of the algebraic problem of orthogonal designs to orders a power of 2 51

§3.14. Solutions of the algebraic problem of orthogonal designs in orders 4t and 8t (t odd) 55

§3.15. Solution of the algebraic problem of orthogonal designs in orders 16t (t odd) 61

§3.16. Solution of the algebraic problem of orthogonal designs in order 32t (t odd) 67

§3.17. The Periodicity Theorem and the general solution of the algebraic problem of orthogonal designs 68

§3.18. Combining the algebraic solution with combinatorial facts 70

Chapter 4: General Constructions for Orthogonal Designs 73

§4.1. Some orthogonal designs exist 73

§4.2. Some matrix results and constructions for Hadamard matrices W(h,h) and weighing matrices W(h,h-1) 79

§4.3. The Goethals-Seidel array and other constructions using circulant matrices--constraints on constructions using circulant matrices 96

§4.4. Baumert-Hall arrays 120

§4.5. Plotkin arrays 127

§4.6. Golay sequences and other sequences with zero auto-correlation functions--constructions using complementary sequences to form orthogonal designs and weighing matrices 129

§4.7. Existence of weighing matrices 149

§4.8. More specific constructions using circulant matrices 155

§4.9. Using cyclotomy to construct orthogonal designs 171

§4.10. Generalized Goethals-Seidel arrays 181

§4.11. Balanced weighing matrices 190

§4.12. Negacyclic Matrices 205

Chapter 5: Amicable Orthogonal Designs 211

§5.1. Introduction 211

§5.2. Definitions and elementary observations 212

§5.3. The number of variables in an amicable orthogonal design 220

§5.4. The algebraic theory of amicable orthogonal designs 227

§5.5. The combinatorial theory of amicable orthogonal designs 231

§5.6. Construction of amicable orthogonal designs 240

§5.7. Amicable Hadamard matrices 252

§5.8. Constructions using amicable orthogonal designs--product designs and Hadamard matrices 261

§5.9. Using families of matrices-repeat designs 273

§5.10. Structure of amicable weighing matrices 281

§5.11. A generalization of amicability-families 283

Chapter 6: Robinson's Theorem 289

Chapter 7: The Existence of Hadamard Matrices and Asymptotic Existence Results for Orthogonal Designs 299

§7.1. Existence of Hadamard matrices 300

§7.2. Asymptotic existence results for orthogonal designs 305

§7.3. n-tuples of the form $(2^{p}b_1, 2^{p}b_2, \ldots, 2^{p}b_n)$ 312

Chapter 8: Results on the Existence Conjectures--Numerical Results--Unsolved Problems 323

§8.1. $W(n, k^2)$ for n odd 323

§8.2. Summary of unsolved problems and known results on the existence of $W(n, k^2+\ell^2)$ and orthogonal designs for $n \equiv 2 \pmod 4$ 326

§8.3. Results on orthogonal designs (1,k) and skew-weighing matrices; The weighing matrix conjecture for orders $\equiv 0 \pmod 4$ 335

Appendices: 347

A. Order 12: First rows to use in the Goethals-Seidel array 348

B. Order 20: First rows to use in the Goethals-Seidel array 349

C. Order 28: First rows to use in the Goethals-Seidel array 352

D. Orthogonal designs in powers of 2, especially orders 16, 32 and 64 358

E. Orthogonal designs in order 24 373

F. Orthogonal designs in orders 40, 80 379

G. Results, by order, on orthogonal designs in orders $n \equiv 0 \pmod 4$ 386

H. Some complementary sequences 400

I. Product designs 408

J. Designs which may exist in order $n \equiv 0 \pmod 4$ 411

K. The smallest t such that an Hadamard matrix of order $2^t q$ exists for $q(\text{odd}) < 10,000$ 415

L. Amicable orthogonal designs in order 12 426

Problems 429

References 439

Index 457