

	Page
<u>Preface</u>	
<u>Part I: The Synthetic Theory</u>	
Introduction	1
1. Basic structure on the geometric line	2
2. Differential calculus	9
3. Higher Taylor formulae (one variable)	13
4. Partial derivatives	16
5. Higher Taylor formulae in several variables. Taylor series	21
6. Some important infinitesimal objects	25
7. Tangent vectors and the tangent bundle	33
8. Vector fields and infinitesimal transformations	39
9. Lie bracket - commutator of infinitesimal transformations	45
10. Directional derivatives	50
11. Some abstract algebra and functional analysis. Application to proof of Jacobi identity	57
12. The comprehensive axiom	61
13. Order and integration	69
14. Forms and currents	74
15. Currents defined using integration. Stokes' Theorem	83
16. Weil algebras	88
17. Formal manifolds	99
18. Differential forms in terms of 1-neighbour simplices	108
19. Open covers	117
20. Differential forms as quantities	124
21. Pure Geometry	129

## PART II: Categorical Logic

Introduction	133
1. Generalized elements	135
2. Satisfaction (1)	138
3. Extensions and descriptions	144
4. Semantics of function objects	151
5. Axiom 1 revisited	158
6. Comma categories	161
7. Dense class of generators	169
8. Satisfaction (2), and topological density	173
9. Geometric theories	179

## PART III: Models

Introduction	182
1. Models for Axioms 1, 2, and 3	183
2. Models for $\varepsilon$ -stable geometric theories	192
3. Axiomatic theory of well-adapted models (1)	200
4. Axiomatic theory of well-adapted models (2)	208
5. The algebraic theory of smooth functions	216
6. Germ-determined $\mathbb{T}_0$ -algebras	229
7. The open cover topology	237
8. Construction of well-adapted models	244
9. $W$ -determined algebras, and manifolds with boundary	252
10. A field property of $R$ , and the synthetic role of germ algebras	267
11. Order and integration in the Cahiers topos	276
<u>Loose ends</u>	285
<u>Historical remarks</u>	288
<u>Appendix A: Functorial semantics</u>	295
<u>Appendix B: Grothendieck topologies</u>	300
<u>Appendix C: Cartesian closed categories</u>	303
<u>References</u>	304
<u>Index</u>	310