

Contents

Introduction

Chapter I

Holomorphic vector bundles and the geometry of \mathbb{P}_n	1
§ 1. Basic definitions and theorems	1
1.1. Serre duality, the Bott formula, Theorem A and Theorem B	1
1.2. Chern classes and dual classes	12
§ 2. The splitting of vector bundles	21
2.1. The theorem of Grothendieck	22
2.2. Jump lines and the first examples	26
2.3. The splitting criterion of Horrocks	39
2.4. Historical remarks	44
§ 3. Uniform bundles	46
3.1. The standard construction	46
3.2. Uniform r -bundles over \mathbb{P}_n , $r < n$	51
3.3. A non-homogeneous uniform $(3n-1)$ -bundle over \mathbb{P}_n	62
3.4. Some historical remarks, further results and open questions	70
§ 4. Examples of indecomposable $(n-1)$ -bundles over \mathbb{P}_n	73
4.1. Simple bundles	74
4.2. The null correlation bundle	76
4.3. The example of Tango	81
4.4. Concluding remarks and open questions	88

§ 5. Holomorphic 2-bundles and codimension 2 locally complete intersections	90
5.1. Construction of 2-bundles associated to a locally complete intersection	90
5.2. Examples	101
5.3. Historical remarks	110
§ 6. Existence of holomorphic structures on topological bundles	111
6.1. Topological classification of bundles over \mathbb{P}_n	111
6.2. 2-bundles over \mathbb{P}_2	117
6.3. 2-bundles over \mathbb{P}_3	122
6.4. 3-bundles over \mathbb{P}_3	130
6.5. Concluding remarks	137
Chapter II	
Stability and moduli spaces	139
§ 1. Stable bundles	139
1.1. Some useful results from sheaf theory	139
1.2. Stability: definitions and elementary properties	160
1.3. Examples of stable bundles	179
1.4. Further results and open questions	189
§ 2. The splitting behavior of stable bundles	192
2.1. Construction of subsheaves	193
2.2. Applications of the theorem of Grauert and Müllich	209
2.3. Historical remarks, further results and open questions	234

§ 3. Monads	238
3.1. The theorem of Beilinson	239
3.2. Examples	246
3.3. A stable 2-bundle over \mathbb{P}_4	258
3.4. Historical remarks	268
§ 4. Moduli of stable 2-bundles	271
4.1. Construction of the moduli spaces for stable 2-bundles over \mathbb{P}_2	271
4.2. Irreducibility of $M_{\mathbb{P}_2}(0,n)$	320
4.3. Examples	344
4.4. Historical remarks, further results, open problems	366