Contents

Preface v

Introduction, a Survey of Some Topological Concepts . . . 3

1. Theory of Sets. Topological Spaces. 3
2. Questions Related to Curves. 5
3. Polyhedra. 8
4. Coincidences and Fixed Points. 14
5. Vector Fields. 17
6. Integration and Topology. 19

Chapter I. Basic Information about Sets, Spaces, Vectors, Groups

1. Questions of Notation and Terminology. 26
2. Euclidean Spaces, Metric Spaces, Topological Spaces. . . 28
3. Compact Spaces. 34
4. Vector Spaces. 38
5. Products of Sets, Spaces and Groups. Homotopy. . . 40

Problems. 43

Chapter II. Two-dimensional Polyhedral Topology

1. Elements of the Theory of Complexes. Geometric Consideration. 45
2. Elements of the Theory of Complexes. Modulo Two Theory. 50
3. The Jordan Curve Theorem. 61
4. Proof of the Jordan Curve Theorem. 65
5. Some Additional Properties of Complexes. 68
6. Closed Surfaces. Generalities. 72
7. Closed Surfaces. Reduction to a Normal Form. . . . 73

Problems. 84

Chapter III. Theory of Complexes

1. Intuitive Approach. 86
2. Simplexes and Simplicial Complexes. 87
3. Chains, Cycles, Homology Groups. 89
4. Geometric Complexes. 95

5. Calculation of the Betti Numbers. The Euler-Poincaré Characteristic. 99
6. Relation between Connectedness and Homology. . . . 103
7. Circuits. 105
Problems. 107

Chapter IV. Transformations of Complexes. Simplicial Approximations and Related Questions

1. Set-transformations. Chain-mappings. 110
2. Derivation. 112
3. The Brouwer Fixed Point Theorem. 117
4. Simplicial Approximation. 119
5. The Brouwer Degree. 124
6. Hopf's Classification of Mappings of n-spheres on n-spheres. 132
7. Some Theorems on the Sphere. 134
Problems. 140

Chapter V. Further Properties of Homotopy. Fixed Points. Fundamental Group. Homotopy Groups

1. Homotopy of Chain-mappings. 142
2. Homology in Polyhedra. Relation to Homotopy. . . . 148
3. The Lefschetz Fixed Point Theorem for Polyhedra. . . . 153
4. The Fundamental Group. 157
5. The Homotopy Groups. 170
Problems. 180

Chapter VI. Introduction to Manifolds. Duality Theorems

1. Differentiable and Other Manifolds. 183
2. The Poincaré Duality Theorem. 188
3. Relative Homology Theory. 195
4. Relative Manifolds and Related Duality Theory (Elementary Theory). Alexander's Duality Theorem. 202
Problems. 206

Bibliography 208

List of Symbols 211

Index 213