

CONTENTS

Chapter I. Vector measures

§ 1. Classes of sets	1
1. Clans	1
2. Tribes	2
3. Semi-tribes	3
4. Semi-clans and lattices	7
5. Monotone classes	9
6. The classes $\sigma(\mathcal{A})$	12
§ 2. Set functions	14
1. Additive set functions	14
2. Positive additive set functions on a clan	15
3. Countably additive set functions	17
4. Measures	18
5. Positive measures	20
6. Operator set functions	22
7. Complex set functions	23
8. Uniqueness of the set functions	23
9. Atomic measures	25
§ 3. Variation of set functions	32
1. Definition of the variation	32
2. Properties of the variation	33
3. Variation of the scalar additive set functions	38
4. Set functions with finite variation	40
5. Locally bounded set functions	42
6. Variation of the scalar measures on a semi-tribe	44
§ 4. Semi-variation of set functions	50
1. Definition of the semi-variation	51
2. Properties of the semi-variation	52
3. Semi-variation of set functions with values in a conjugate space	54
4. Set functions with finite semi-variation	56
§ 5. Extension of set functions	57
1. Extension of additive set functions	57
2. Completion of an additive set function	59
3. Extension of set functions with finite variation	60
4. Extension of positive measures	64
5. Jordan measure	78

Chapter II. Integration

§ 6. Measurable functions	82
1. Step functions	82
2. Totally measurable functions	83
3. Real functions measurable with respect to a tribe	83
4. Sequences of measurable real functions	87
5. Measurable functions with respect to a measure	89
6. Sequences of μ -measurable functions	94
7. Simply measurable operator functions	101
8. Weakly measurable operator functions	104
§ 7. Integration of step functions	106
1. Definition and properties	106
2. Convergence in mean on the space $\mathcal{E}_E(\mathcal{C})$	110
3. Cauchy sequences of step functions	112
§ 8. Integrable functions	119
1. Definition and properties	120
2. Convergence in mean on the space \mathcal{L}_E^1	127
3. Criteria of integrability	133
4. The space L^1	138
§ 9. The spaces \mathcal{M}_E^∞ and \mathcal{L}_E^∞	139
1. Integration of totally measurable functions	139
2. Linear operations on the space $\mathcal{M}_E(\mathcal{C})$	142
3. Dominated operations	149
4. The semi-norm N_∞	153
5. Almost totally measurable functions	154
6. Operations on $\mathcal{M}_E^\infty(\mathcal{C})$	156
7. The space $\mathcal{L}_E^\infty(\mu)$	161
§ 10. Measures defined by densities	163
1. Locally integrable functions	163
2. Measures defined by densities	164
3. Integration with respect to a positive measure defined by density	165
4. Integration with respect to a vector measure defined by density	169
5. Properties of measures defined by densities	173
6. Absolutely continuous measures	176
7. Measures with the direct sum property	179
8. The theorem of Lebesgue-Nikodym	182
9. Further properties of the measures defined by densities	186
10. Singular measures	188
11. Conditional expectations	190
12. Martingales	192
13. Convergence theorems for martingales	193

§ 11. The lifting property of the space \mathcal{L}^∞	199
1. Definition and properties	199
2. Lifting on sets	201
3. Linear liftings	203
4. The existence of the lifting	206
5. Limits of measurable functions	209
6. Functions with the lifting property	211
§ 12. The spaces \mathcal{L}_E^p	217
1. Definition and properties	217
2. The inequalities of Hölder and Minkowski	219
3. Convergence in mean of order p	222
4. Computation of the semi-norm N_p	228
5. Relations between the spaces \mathcal{L}_E^p	237
§ 13. Linear operations on \mathcal{L}_E^p	241
1. The q -variation	241
2. The q -semi-variation	246
3. Linear operations on \mathcal{L}_E^p	255
4. The generalized theorem of Lebesgue-Nikodym	263
5. Integral representation of linear operations on \mathcal{L}_E^p	279
Chapter III. Regular measures	
§ 14. Borel sets. Borel measures	287
1. The semi-tribe of the relatively compact Borel sets	287
2. The tribe of the Borel sets	290
3. Baire sets	291
4. Borel measures. Baire measures	297
5. Integration with respect to a Borel measure	298
§ 15. Regular measures	301
1. Topologisation of $\mathcal{P}(T)$	302
2. Regular set functions	302
3. Regular additive set functions on a clan	304
4. Set functions regular on a subclass	307
5. Regular positive set functions	309
6. Regular set functions with finite variation	313
7. Integration with respect to regular measures	320
8. Measurable functions	334
§ 16. Construction and extension of regular measures	339
1. Extension of regular set functions	339
2. Construction of regular positive Borel measures	347
3. Construction of regular Borel vector measures	351
4. Regularity of the vector measures on a subclass	354

§ 17. Stieltjes measures on the real line	356
1. Functions with finite variation	356
2. Stieltjes measures deduced from functions with finite variation	358
3. Functions with finite variation deduced from Stieltjes measures	363
§ 18. Reduction of the integration on abstract spaces to the integration on locally compact spaces	368
1. Theorem of Stone	368
2. Theorem of Kakutani	372
§ 19. Integral representation of linear operations on $\mathcal{K}_E(T)$	374
1. The space $\mathcal{K}_E(T)$	374
2. Linear operations on $\mathcal{K}_E(T)$	375
3. Dominated operations	379
4. Positive functionals on $\mathcal{K}(T)$	390
5. Continuous linear operations	398
§ 20. Disintegration of measures	402
1. Images of measures	402
2. The strong lifting property	406
3. Disintegration of measures	408
Notes and remarks	413
Bibliography	417
Index of notations	427
Index	429
Other titles published in this series	433