## Contents

Part	<del></del>	
ine	e Basic Subject Matter	1
Cha	pter 1	
Affi	ne Group Schemes	3
1.1	What We Are Talking About	3
1.2	Representable Functors	
1.3	Natural Maps and Yoneda's Lemma	4 5 7 9
1.4	Hopf Algebras	7
1.5	Translating from Groups to Algebras	
1.6	Base Change	11
Chaj	pter 2	
Affi	ne Group Schemes: Examples	13
2.1	Closed Subgroups and Homomorphisms	13
2.2	Diagonalizable Group Schemes	14
2.3	Finite Constant Groups	16
2.4	Cartier Duals	16
Cha	pter 3	
Rep	presentations	21
3.1	Actions and Linear Representations	21
3.2	Comodules	22
3.3	Finiteness Theorems	24
3.4	Realization as Matrix Groups	25
3.5	Construction of All Representations	25

V111			٠	٠
	v	1	1	1

	apter 4 gebraic Matrix Groups	28
4.1	Cloud Catalia III	
4.1	Closed Sets in k <sup>n</sup>	28
4.3	Algebraic Matrix Groups Matrix Groups and Their Closures	29
4.4	From Closed Sets to Functors	30
4.5	Rings of Functions	30
4.6	Diagonalizability	32
	- ingonunization (	33
Par	rt II	
De	ecomposition Theorems	37
	pter 5	
Irre	educible and Connected Components	39
5.1	Irreducible Components in $k^n$	39
5.2	Connected Components of Algebraic Matrix Groups	40
5.3	Components That Coalesce	41
5.4	Spec A	41
5.5	The Algebraic Meaning of Connectedness	42
5.6	Vista: Schemes	43
Cha	apter 6	
Co	nnected Components and Separable Algebras	46
6.1	Components That Decompose	46
6.2	Separable Algebras	46
6.3	Classification of Separable Algebras	47
6.4	Etale Group Schemes	49
6.5	Separable Subalgebras	49
6.6	Connected Group Schemes	50
6.7	Connected Components of Group Schemes	51
6.8	Finite Groups over Perfect Fields	52
Cha	pter 7	
-	oups of Multiplicative Type	54
7.1	Separable Matrices	54
7.2	Groups of Multiplicative Type	55
7.3	Character Groups	55
7.4	Anisotropic and Split Tori	56
7.5	Examples of Tori	57
7.6	Some Automorphism Group Schemes	58
7.7	A Rigidity Theorem	59

Contents			

ix

Chapter 8	62
Unipotent Groups	02
8.1 Unipotent Matrices	62
8.2 The Kolchin Fixed Point Theorem	62
8.3 Unipotent Group Schemes	63 65
8.4 Endomorphisms of G <sub>a</sub>	66
8.5 Finite Unipotent Groups	
Chapter 9	68
Jordan Decomposition	00
9.1 Jordan Decomposition of a Matrix	68
9.2 Decomposition in Algebraic Matrix Groups	69
9.3 Decomposition of Abelian Algebraic Matrix Groups	69
9.4 Irreducible Representations of Abelian Group Schemes	70 70
9.5 Decomposition of Abelian Group Schemes	70
Chapter 10 Nilpotent and Solvable Groups	73
•	73
10.1 Derived Subgroups	73
10.2 The Lie-Kolchin Triangularization Theorem 10.3 The Unipotent Subgroup	75
<ul><li>10.3 The Unipotent Subgroup</li><li>10.4 Decomposition of Nilpotent Groups</li></ul>	75
10.5 Vista: Borel Subgroups	76
10.6 Vista: Differential Algebra	77
Part III	0.4
The Infinitesimal Theory	81
Chapter 11 Differentials	83
2 more itals	03
11.1 Derivations and Differentials	83 84
11.2 Simple Properties of Differentials	85
11.3 Differentials of Hopf Algebras	86
11.4 No Nilpotents in Characteristic Zero	87
<ul><li>11.5 Differentials of Field Extensions</li><li>11.6 Smooth Group Schemes</li></ul>	88
11.7 Vista: The Algebro-Geometric Meaning of Smoothness	89
11.8 Vista: Formal Groups	89

X Contents

	pter 12	
Lie	Algebras	92
	Invariant Operators and Lie Algebras Computation of Lie Algebras Examples	92 93 95
12.4	Subgroups and Invariant Subspaces Vista: Reductive and Semisimple Groups	96 97
Part Fair	IV thful Flatness and Quotients	101
_	pter 13	
Fait	thful Flatness	103
	Localization Properties	103 104
	Transition Properties Generic Faithful Flatness	105
	Proof of the Smoothness Theorem	106 107
-	pter 14	
Fait	thful Flatness of Hopf Algebras	109
14.1	Proof in the Smooth Case	109
14.2 14.3	Proof with Nilpotents Present Simple Applications	110 111
14.4	F	112
	oter 15	
Quo	otient Maps	114
15.1	_	114
15.2 15.3		115 115
15.4	,	115
15.5	Sheaf Property of Quotients	116
15.6 15.7	Coverings and Sheaves Vista: The Etale Topology	117 118
Chap	oter 16	
Con	struction of Quotients	121
16.1	Subgroups as Stabilizers	121
16.2 16.3	Difficulties with Coset Spaces	122
16.4	Construction of Quotients Vista: Invariant Theory	123 125

Contents	x

Part Desc	V cent Theory	129
Chap	ter 17	
-	cent Theory Formalism	131
17.1	Descent Data	131
17.2	The Descent Theorem	132
17.3	Descent of Algebraic Structure	133
17.4	Example: Zariski Coverings	134
17.5		134
17.6	Twisted Forms and Cohomology	135
17.7	Finite Galois Extensions	136
17.8		138
Chap	ter 18	
Desc	cent Theory Computations	140
18.1	A Cohomology Exact Sequence	140
18.2	Sample Computations	141
18.3		142
18.4	Principal Homogeneous Spaces and Cohomology	142
18.5	Existence of Separable Splitting Fields	144
18.6	Example: Central Simple Algebras	145
18.7	Example: Quadratic Forms and the Arf Invariant	147
18.8	Vanishing Cohomology over Finite Fields	148
App	endix: Subsidiary Information	151
A.1	Directed Sets and Limits	151
A.2	Exterior Powers	152
<b>A.3</b>	Localization, Primes, and Nilpotents	152
<b>A.4</b>	Noetherian Rings	153 153
A.5	The Hilbert Basis Theorem	153
A.6	The Krull Intersection Theorem	155
<b>A</b> .7	The Noether Normalization Lemma	155
<b>A.8</b>	The Hilbert Nullstellensatz	156
<b>A.9</b>	Separably Generated Fields	156
A.10	Rudimentary Topological Terminology	130
Fur	ther Reading	158
Ind	ex of Symbols	161
Index		