

Contents

PREFACE	v
1. THE ALGEBRA OF VECTORS	1
1.1 Scalars	1
1.2 Vectors	2
1.3 Linear dependence	6
1.4 Basis vectors	8
1.5 Applications	10
(a) Equation of a straight line	10
(b) Mass centre of a system of particles	10
(c) Relative velocity	11
(d) Forces	11
1.6 Worked examples and problems	12
2. PRODUCTS OF VECTORS	19
2.1 Multiplying vectors	19
2.2 The scalar product	19
2.3 The vector product	21
2.4 Triple products	22
2.5 Applications	24
(a) Equation of a plane	24
(b) Angular velocity	25
(c) Angular momentum	26
(d) Moment of a force	27
2.6 Worked examples and problems	27
3. DIFFERENTIATION	34
3.1 The vector ordinary derivative	34
3.2 Vector partial derivatives	36
3.3 Differentials	38
3.4 Applications	40
(a) Velocity and acceleration	40
(b) Newton's law of motion for a particle	40
(c) Rotating frames of reference	41
(d) Curvature	42
3.5 Worked examples and problems	43

4. INTEGRATION	50
4.1 The integral of a vector	50
4.2 Line integrals	51
4.3 Surface integrals	52
4.4 Volume integrals	53
4.5 Applications	54
(a) Work done by a force	54
(b) Solid angle	54
(c) Flux of fluid through a surface	56
(d) Angular momentum of a spinning rigid body	56
4.6 Worked examples and problems	57
5. VECTOR OPERATORS	64
5.1 Gradient, divergence and curl	64
5.2 Properties involving nabla	65
5.3 Directional derivatives	67
5.4 Scalar and vector potentials	70
5.5 Applications	71
(a) Potential of gravitating particles	71
(b) Tangent plane to a surface	72
(c) Rotation of a rigid body. Vorticity	73
(d) Magnetic vector potential	73
5.6 Worked examples and problems	74
6. INTEGRATION THEOREMS	80
6.1 Line-surface theorems	80
6.2 Surface-volume theorems	86
6.3 Invariance of grad, div and curl	89
6.4 Conservative fields	90
6.5 Applications	91
(a) Mass conservation in a fluid	91
(b) Fluid vortex tubes and circulation	92
6.6 Worked examples and problems	93
7. ORTHOGONAL CURVILINEAR COORDINATES	101
7.1 Curvilinear Coordinates	101
7.2 Expressions for div, grad and curl	102
7.3 Applications	106
(a) Cylindrical polar coordinates	106
(b) Spherical polar coordinates	107
7.4 Worked examples and problems	109
ANSWERS TO PROBLEMS	115
INDEX	119