

Table of Contents

Introduction	1
Chapter 0 Notation, Conventions, Preliminaries	7
Chapter I The Browder-Livesay Invariants	10
I.1 Involutions of Spheres	10
I.1.1 Desuspensions and Characteristic Submanifolds	10
I.1.2 Equivariant Surgery	11
I.1.3 The Desuspension Theorem	12
I.1.4 Concordance of Desuspensions	14
I.2 Involutions of Simply Connected Manifolds	14
I.2.1 Characteristic Submanifolds and h -regularity	14
I.2.2 The Browder-Livesay Invariants	15
Chapter II Realization of the Browder-Livesay Invariants	18
II.1 The Realization Theorem	18
II.2 Constructions with Involutions	18
II.2.1 $M \cup_T M^*$	18
II.2.2 $(T, M^n) \# N$	19
II.3 Proof of Theorem II.1-D	20
II.4 Proof of Theorem II.1-A	21
II.5 Homology 3-Spheres	24
II.6 Manifolds with the Same Regular Homotopy Type as Line Bundles over Projective Spaces	26
II.7 Invariant Codimension 2 Spheres	26
Chapter III Relations with Non-simply-connected Surgery Ob- structions	28
III.1 Normal Invariants	28
III.1.1 Normal Maps, Cobordisms and Invariants	28
III.1.2 $G/PL, G/O$	29
III.1.3 Homotopy Triangulations and Smoothings	30

III.2	Non-simply-connected Surgery Obstructions	31
III.2.1	The Groups $L_n(\pi, \omega)$	31
III.2.2	The Obstruction Groups for $\pi = \mathbf{Z}_2$	33
III.3	Relations with the Browder-Livesay Invariants.	34
III.3.1	$l_n: BL_n(\varepsilon) \rightarrow L_{n-1}(\mathbf{Z}_2, -\varepsilon)$	34
III.3.2	Ambient vs. Abstract Surgery Obstructions.	36
III.3.3	Proof of Lemma 2, II.4	37
III.3.4	Proof of Theorem II.1-C	38
Chapter IV	Combinatorial Classification of Involutions	39
IV.1	Some Maps and Exact Sequences	39
IV.2	Computation of $[P^n, G/PL]$	40
IV.3	The Classification Theorem	43
IV.3.1	Homotopy Projective Spaces	43
IV.3.2	Suspension	44
IV.3.3	Two Useful Theorems	45
IV.3.4	The Classification Theorem.	48
IV.3.5	Proof of Theorem II.1-B (p.l. case)	49
IV.4	Another Relation with Non-simply-connected Surgery Obstructions	49
IV.4.1	$\sigma = \pm \tau$	49
IV.4.2	On Theorem II.1-C	52
IV.5	On the Topological Classification of Involutions	53
Chapter V	Smooth Involutions	55
V.1	General Remarks on Smooth Involutions of Spheres	55
V.2	Normal Invariants and Browder-Livesay Invariants	56
V.2.1	Involutions of Spheres	56
V.2.2	Involutions of Simply-connected Manifolds	57
V.2.3	$\sigma = \pm \tau$ again	60
V.3	Differentiable Structure of Spheres and Other Double Coverings	62
V.4	Proof of Theorem II.1-B, Smooth Case	63
V.5	Involutions and the Generalized Kervaire Invariant	65
V.5.1	The Generalized Kervaire Invariant	65
V.5.2	Odd Multiples of an Involution	66

V.6	Smooth Involutions of Spheres of Low Dimension	69
V.6.1	Involutions of 7-Spheres	69
V.6.2	Spheres that Admit Involutions	70
V.7	Action of $\theta^n(\partial\pi)$ (After Browder)	73
V.8	How to Prove $\sigma=\tau$	74
Chapter VI	Codimension 2 Invariant Spheres	78
VI.1	Invariant vs. Characteristic Spheres	78
VI.2	Applications	80
VI.2.1	A Non-embedding Result	80
VI.2.2	Some Brieskorn-Hirzebruch Involutions	80
VI.3	Knotted and Unknotted Codimension 2 Invariant Spheres	81
VI.4	Cobordism Classes of Invariant Knots.	91
Some Unsolved Problems	95
References	97
Index	101
List of Symbols	103