

Preface

1: SOME OLD TOPOLOGICAL PROBLEMS	1
1.0. Introduction	1
1.1. The Descartes-Euler Theorem	2
1.2. Coloring Maps and Graphs	9
1.3. The Jordan Curve Theorem; Knots	19
1.4. Further Early Investigations	22
2: THE GEOMETRY OF COMPLEXES	25
2.1. What is Topology	25
2.2. Simplicial Complexes and Maps; Polyhedra	31
2.3. Abstract Simplicial Complexes; Cutting and Pasting . .	46
2.4. Historical Comments	52
3: THE CLASSIFICATION OF SURFACES	55
3.0. Introduction	55
3.1. The Definition of a Surface	55
3.2. Representing Surfaces by Symbols	60
3.3. The Normal Form for Some Surfaces	78

3.4.	The Classification Theorems	82
3.5.	Bordered and Noncompact Surfaces	96
3.6.	Historical Comments	102
4:	THE HOMOLOGY GROUPS	107
4.1.	Some Motivation	107
4.2.	The Orientation of a Simplex	110
4.3.	The Definition of the Homology Groups	115
4.4.	The Homology Groups of a Cone	128
4.5.	The Topological Invariance of Homology Groups	133
4.6.	Historical Comments	135
5:	MAPS AND HOMOTOPY	139
5.1.	Simplicial Maps Again	139
5.2.	Homotopy	146
5.3.	Simplicial Approximations	150
5.4.	The Barycentric Subdivision	155
5.5.	The Simplicial Approximation Theorem; Induced Homomorphisms	164
5.6.	Historical Comments	168
6:	FIRST APPLICATIONS OF HOMOLOGY THEORY	169
6.1.	A Quick Review	169
6.2.	Local Homology Groups	174
6.3.	Some Invariance Theorems	182
6.4.	Homology with Arbitrary Coefficients; The Mod 2 Homology Groups	190
6.5.	Pseudomanifolds; Orientability	195
6.6.	Euler's Theorem Revisited	205
6.7.	Retracts and the Brouwer Fixed-Point Theorem	214
6.8.	Historical Comments	219

7: MAPS OF SPHERES AND MORE APPLICATIONS	225
7.1. The Degree of a Map; Vector Fields	225
7.2. The Borsuk-Ulam Theorem and the Ham Sandwich Problem	234
7.3. More on Degrees of Maps; Zeros of Polynomials	248
7.4. Local Degrees, Solvability of Equations and Some Complex Analysis	253
7.5. Extending Maps	275
7.6. The Jordan Curve Theorem and Other Separation Theorems	283
7.7. Historical Comments	289
8: CONCLUDING REMARKS	297
APPENDIX A: The Topology of \mathbb{R}^n	305
APPENDIX B: Permutations and Abelian Groups	311
APPENDIX C: The Incidence Matrices	323
List of Symbols	331
Bibliography	337
Index	355