

# TABLE OF CONTENTS

CHAPTER ONE. INTRODUCTION	1
A. Notation and definitions.....	2
B. Some examples of linking.....	4
CHAPTER TWO. CODIMENSION ONE AND OTHER MATTERS	8
A. Knots in the plane.....	9
B. The Jordan curve theorem and chord theorem.....	13
C. Knots in the torus.....	17
D. The mapping class group of the torus.....	26
E. Solid tori.....	29
F. Higher dimensions.....	33
G. Connected sum and handlebodies.....	39
CHAPTER THREE. THE FUNDAMENTAL GROUP	47
A. Knot and link invariants.....	47
B. The knot group.....	51
C. Torus knots.....	53
D. The Wirtinger presentation.....	56
E. Regular projections.....	63
F. Computations for links.....	65
G. Chains.....	70
H. Iterated chains and Antoine's necklace.....	73
I. Horned spheres.....	76
J. Application of $\pi_1$ to higher-dimensional knots.....	83
K. Unsplittable links in 4-space.....	88
L. Generalized spinning.....	96
CHAPTER FOUR. THREE-DIMENSIONAL PL GEOMETRY	100
A. Three theorems of Papakyriakopoulos.....	100
B. The unknotting theorem.....	103
C. Knotting of tori in $S^3$ .....	106
D. Knots in solid tori and companionship.....	110
E. Applications of the sphere theorem.....	116
CHAPTER FIVE. SEIFERT SURFACES	118
A. Surfaces and genus.....	118
B. Higher-dimensional Seifert surfaces.....	127
C. Construction of the cyclic coverings of a knot complement using Seifert surfaces.....	128

D. Linking numbers.....	132
E. Boundary linking.....	137

## CHAPTER SIX. FINITE CYCLIC COVERINGS AND TORSION INVARIANTS

A. Torsion numbers.....	145
B. Calculation using Seifert surfaces.....	146
C. Calculation using surgery in $S^3$ .....	152
D. Surgery description of knots.....	158

## CHAPTER SEVEN. INFINITE CYCLIC COVERINGS AND THE ALEXANDER INVARIANT

A. The Alexander invariant.....	160
B. Seifert surfaces again.....	163
C. Surgery again.....	168
D. Computing the Alexander invariant from the knot group.....	174
E. Additivity of the Alexander invariant.....	179
F. Higher-dimensional examples: plumbing.....	180
G. Nontrivial knots in higher dimensions with group $Z$ .....	185
H. Higher-dimensional knots with specified polynomial.....	187
I. Alexander invariants of links.....	190
J. Brunnian links in higher dimensions.....	197

## CHAPTER EIGHT. MATRIX INVARIANTS

A. Seifert forms and matrices.....	200
B. Presentation matrices.....	203
C. Alexander matrices and Alexander polynomials.....	206
D. The torsion invariants.....	212
E. Signature and slice knots.....	216
F. Concordance.....	227

## CHAPTER NINE. 3-MANIFOLDS AND SURGERY ON LINKS

A. Introduction.....	233
B. Lens spaces.....	233
C. Heegaard diagrams.....	239
D. The Poincaré conjecture, homology spheres and Dehn's construction.....	244
E. A theorem of Bing.....	251
F. Surgery on 3-manifolds.....	257
G. Surgery instructions in $R^3$ or $S^3$ .....	258
H. Modification of surgery instructions.....	264
I. The fundamental theorem of Lickorish and Wallace.....	273
J. Knots with property $P$ .....	280

CHAPTER TEN. FOLIATIONS, BRANCHED COVERS, FIBRATIONS AND  
SO ON

A. Foliations.....	284
B. Branched coverings.....	292
C. Cyclic branched covers of $S^3$ .....	297
D. Cyclic coverings of $S^3$ branched over the trefoil ( a lengthy example).....	304
E. The ubiquitous Poincaré homology sphere.....	308
F. Other branched coverings of $S^3$ .....	312
G. Arbitrary 3-manifolds as branched coverings of $S^3$ ..	319
H. Fibred knots and links.....	323
I. Fiberings the complement of a trefoil.....	327
J. Constructing fibrations.....	335
K. Open book decompositions.....	340

CHAPTER ELEVEN. A HIGHER-DIMENSIONAL SAMPLER

A. Forming knots by adding handles.....	342
B. Trivial sphere pairs contain nontrivial ball pairs..	345
C. The Smith conjecture.....	347
D. Kervaire's characterization of knot groups.....	350
E. Contractible 4-manifolds.....	355

APPENDIX A. COVERING SPACES AND SOME ALGEBRA IN A NUTSHELL 3

APPENDIX B. DEHN'S LEMMA AND THE LOOP THEOREM	374
---	-----

APPENDIX C. TABLE OF KNOTS AND LINKS	388
--------------------------------------	-----

REFERENCES	429
------------	-----

INDEX	437
-------	-----