Contents

Notation xv

Chapter I. Preliminaries 1

1. Fundamental Notions of Set Theory 1
2. Relations and Mappings 4
3. Partial and Linear Orderings; Cartesian Products 8
4. Lattices 10
5. Algebraic Structures 14
6. Categories and Functors 17

Chapter II. Topological Spaces 21

7. Open and Closed Sets 21
8. Topologies and Neighborhoods 25
9. Limit Points 26
10. Bases and Subbases 30
11. First and Second Countable Spaces 32
12. Metric Spaces 33
13. Nets 36
14. Filters 43
15. Topologies Defined by Other Topologies 46

Examples and Exercises 49

Chapter III. Continuity and Separation Axioms 53

16. Continuous and Open Mappings 53
17. Topologies Defined by Mappings 60
18. Separation Axioms 60
19. Continuous Functions on Normal Spaces 68

Examples and Exercises 74

Chapter IV. Methods for Constructing New Topological Spaces from Old 77

20. Subspaces 77
21. Topological Sums 80
22. Topological Products 83
23. Quotient Topology and Quotient Spaces 90
24. Projective and Inductive Limits 95
Examples and Exercises 99

Chapter V. Uniform Spaces 103

25. Uniformities and Topologies 103
26. Uniformity and Separation Axioms 108
27. Uniformizable Spaces 111
28. Uniform Continuity and Uniform Spaces 113
29. Completeness in Uniform Spaces 117
30. Completeness, Compactness, and Completions 120
31. Topological Groups and Topological Vector Spaces 126
32. Metrizability 131
33. Fixed Points 136
34. Proximity Spaces 141
Examples and Exercises 144

Chapter VI. Compact Spaces and Various Other Types of Spaces 149

35. Compact Spaces 149
36. Countable Compactness and Sequential Compactness 157
37. Compactness in Metric Spaces 159
38. Locally Compact Spaces 163
39. *MB*-Spaces 168
40. k-Spaces and k_r-Spaces 171
41. Baire Spaces 175
42. Pseudocompact Spaces 177
43. Paracompact Spaces 180
44. Compactifications 186
Examples and Exercises 190

Chapter VII. Generalizations of Continuous Maps 195

45. Almost Continuous Maps 195
46. Closed Graphs 199
47. Almost Continuity and Closed Graphs 203
48. Graphically Continuous Maps 207
49. Nearly Continuous and w-Continuous Maps 210

50. Semicontinuous Maps 216
51. Approximately Continuous Functions 220
52. Applications of Almost Continuity 222
Examples and Exercises 232

Chapter VIII. Function Spaces **237**

53. The Set of All Maps 237
54. Compact–Open Topology and the Topology of Joint Continuity . . 240
55. Subsets of F^E with Induced Topologies 242
56. The Uniformities on F^E 244
57. $\mathfrak{S}$-Uniformities and $\mathfrak{S}$-Topologies 248
58. Equicontinuous Maps 250
59. Equicontinuity and Metric Spaces 253
60. Sequential Convergence in Function Spaces 261
Examples and Exercises 263

Chapter IX. Extensions of Mappings **267**

61. Extensions of Maps on Completely Regular and Metric Spaces . . . 267
62. The Hahn–Banach Extension Theorem 274
63. A General Extension Theorem 283
Examples and Exercises 286

Chapter X. $C(X)$ Spaces **289**

64. Stone–Weierstrass Theorem 289
65. Embeddings of X into $C(X)$ 296
66. $C(X)$ Spaces for Compact Spaces X 301
67. Separability in $C(X)$ 305
68. $C(X)$ Spaces for Completely Regular Spaces X 308
69. Characterization of Banach and Fréchet Spaces $C(X)$ 310
70. Characterization of Locally Convex Spaces $C(X)$ 314
Epilogue 322
Examples and Exercises 322

BIBLIOGRAPHY 327
INDEX 333