

Chapter 0

Calculus in Euclidean Space	1
0.1 Euclidean Space	1
0.2 The Topology of Euclidean Space	2
0.3 Differentiation in \mathbb{R}^n	3
0.4 Tangent Space	5
0.5 Local Behavior of Differentiable Functions (Injective and Surjective Functions)	6

Chapter 1

Curves	8
1.1 Definitions	8
1.2 The Frenet Frame	10
1.3 The Frenet Equations	11
1.4 Plane Curves; Local Theory	15
1.5 Space Curves	17
1.6 Exercises	20

Chapter 2

Plane Curves: Global Theory	21
2.1 The Rotation Number	21
2.2 The Umlaufsatz	24
2.3 Convex Curves	27
2.4 Exercises and Some Further Results	29

Chapter 3	
Surfaces: Local Theory	33
3.1 Definitions	33
3.2 The First Fundamental Form	35
3.3 The Second Fundamental Form	38
3.4 Curves on Surfaces	43
3.5 Principal Curvature, Gauss Curvature, and Mean Curvature	45
3.6 Normal Form for a Surface, Special Coordinates	49
3.7 Special Surfaces, Developable Surfaces	54
3.8 The Gauss and Codazzi–Mainardi Equations	61
3.9 Exercises and Some Further Results	66
Chapter 4	
Intrinsic Geometry of Surfaces: Local Theory	73
4.1 Vector Fields and Covariant Differentiation	74
4.2 Parallel Translation	76
4.3 Geodesics	78
4.4 Surfaces of Constant Curvature	83
4.5 Examples and Exercises	87
Chapter 5	
Two-dimensional Riemannian Geometry	89
5.1 Local Riemannian Geometry	90
5.2 The Tangent Bundle and the Exponential Map	95
5.3 Geodesic Polar Coordinates	99
5.4 Jacobi Fields	102
5.5 Manifolds	105
5.6 Differential Forms	111
5.7 Exercises and Some Further Results	119
Chapter 6	
The Global Geometry of Surfaces	123
6.1 Surfaces in Euclidean Space	123
6.2 Ovaloids	129
6.3 The Gauss–Bonnet Theorem	138
6.4 Completeness	144
6.5 Conjugate Points and Curvature	148
6.6 Curvature and the Global Geometry of a Surface	152
6.7 Closed Geodesics and the Fundamental Group	156
6.8 Exercises and Some Further Results	161
References	167
Index	171
Index of Symbols	177