

# TABLE OF CONTENTS

	<u>Page</u>
PREFACE	iii
Chapter A: CONTACT MANIFOLDS AND MAPPING ELEMENT SPACES	1
1. Introduction	1
2. Mappings which Agree to the $r$ -th Order	2
3. A Multilinear Algebraic Formula for Differentials of Maps Between Vector Spaces	6
4. Mapping Element Spaces	9
5. Differentiable Subsets of Manifolds	12
6. Differential Equations and Contact Transformations	13
7. Contact Element Spaces	14
8. The Mapping Element Spaces as Open Subsets of the Contact Spaces	19
9. Prolongation of Group and Lie Algebra Actions	25
ON DIFFERENTIAL INVARIANTS by Sophus Lie	29
Chapter 1: INTRODUCTORY REMARKS	31
1.1	31
1.2 Differential Invariants	32
Chapter 2: FUNCTIONS AND EQUATIONS WHICH ADMIT A FINITE CONTINUOUS GROUP	35
2.1 Origins of Transformation Group Theory	35
2.2 Differential Equations of Invariants	40
2.3 Surfaces which Admit Groups	45
2.4 Relation to Algebraic Invariant Theory	48
Chapter 3: DIFFERENTIAL INVARIANTS OF CONTINUOUS FINITE GROUPS	51
3.1 My Work on Differential Invariants	51
3.2 Halphen's Work on Differential Invariants	55
3.3 The Complete System of Equations for the Differential Invariants	58
3.4 Comments on Chapter 3	63

	<u>Page</u>
Chapter 4: DIFFERENTIAL INVARIANTS OF INFINITE CONTINUOUS GROUPS	69
4.1 Infinite Continuous Groups	69
4.2 Examples	79
4.3 The Group of Volume-Preserving Transformations of the Plane	90
4.4 A General Procedure for Proving Existence of Differential Invariants	98
4.5 Differential Invariants of Secondary, Ordinary Differential Equations	104
4.6 Differential Invariants of Point Trans- formations	110
4.7 Continuation	111
4.8 Higher Order Ordinary Differential Equations	112
4.9 Invariants of a Second Order, Non-Linear Equation	113
4.10 A Group Acting on a Special Type of Second Order, Ordinary Differential Equation	114
4.11 Differential Invariants of Second Order Partial Differential Equations	116
4.12 Invariant Theory of Some Special Second Order Partial Differential Equations	122
4.13 Invariant Theory of Linear Ordinary Differ- ential Equations	124
4.14 Linear Second Order Partial Differential Equations	125
4.15 Curvature as a Differential Invariant	126
4.16 Invariant Theory of First Order Partial Differential Equations	137
4.17 Continuation	138
Chapter 5: CONCLUDING REMARKS	141
5.1 The Equivalence Problem	141
Chapter B: FURTHER WORK IN THE THEORY OF DIFFEREN- TIAL INVARIANTS	149
1. Differential Invariants and Differential Operators	149
2. Algebraic and Differential Invariants of the One-Variable Projective Group	153

	<u>Page</u>
3. The Infinite Lie Group Which Preserves the Class of Linear Differential Operators	169
4. The Schwartzian Derivative as Differential Invariants of the Wilczynski group	176
5. The Method of Semi-Invariants in Invariant Theory	184
 Chapter C: INTEGRAL INVARIANTS	 187
1. Introduction	187
2. Lagrangians for Fiber Spaces	189
3. Lagrangians and Variational Problems for Mapping Element Spaces	190
4. Symmetries (In the Sense of Lie) of Variational Problems	193
5. Symmetries of Lagrangians of the Form $\int L(x,y,y') dx$	195
6. The Cartan Form and the Condition for Lie Symmetry of a Variational Problem	204
 Chapter D: DIFFERENTIAL INVARIANTS AND GROUPS OF SYMMETRIES OF DIFFERENTIAL EQUATIONS	 211
1. Introduction	211
2. Affine Symmetries of Linear, Inhomogeneous Second Order Differential Equations	212
3. The General Lie Theory for Groups of Symmetries of Ordinary Differential Equations	219
4. Integrating Factors for First Order, Ordinary Differential Equations	223
5. A Pfaffian System Framework in Which to Study First Order Differential Equations	227
6. Second Order Ordinary Differential Equations which Admit One-Parameter Groups of Symmetries	232
7. Second Order Ordinary Differential Equations which Admit Two-Dimensional Lie Algebras of Symmetries	238
8. General Remarks About Groups of Symmetries of Second Order Ordinary Differential Equations	243
9. Lie's Problem from Cartan's Exterior Differential System Viewpoint	245

	<u>Page</u>
Chapter E: THE GALOIS-PICARD-VESSIOT THEORY OF LINEAR ORDINARY DIFFERENTIAL EQUATIONS	247
1. Introduction	247
2. The Lie-Vessiot Form of a Linear Ordinary Scalar Differential Equation	248
3. The Galois Group According to the Ideas of Lie and Vessiot	
4. A Galois Theory for a Linear First Order Partial Differential Operator	263
 BIBLIOGRAPHY	 271