

Contents

PREFACE TO THE ENGLISH EDITION	v
PREFACE TO THE GERMAN EDITION	vii

Introduction

1. The Subject	1
2. The Method	2
Table of Several Abbreviations and Symbols	3

Chapter I: Linear Algebra

§1. Modules in Principal Ideal Domains	5
1. Finite Modules	5
2. The Theorem of Elementary Divisors	7
3. Dual Spaces and Complementary Modules	10
4.* Noetherian Rings	11
5.* A Further Basis Theorem	13
§2. Systems of Linear Inequalities	14
1. Minkowski's Point Lattice Theorem	14
2.* Siegel's Proof	16
3. Generalization to Function Fields	18
§3. Linear Divisors	20
1. Basic Concepts	20
2. Norm and Degree of a Linear Divisor	23
3. The Dimension of a Linear Divisor	23
4.* The Riemann-Roch Theorem and the Minkowski Linear Form Theorem	26
§4. Traces, Norms, and Discriminants	27
1. Representations by Matrices	27
2. The Transitivity Formulas	28
3. The Discriminant	29
4. Separable and Inseparable Extensions	30

* The sections marked with an asterisk might be omitted in the first reading.

Appendix to Chapter I: The Theta Function

§1.* The Symplectic Group	32
1. The Basic Properties	32
2. Symplectic Geometry	34
3. The Hyperbolic Plane and Hyperbolic Space	35
4. The Symplectic Modular Group	36
5. The Fundamental Domain	39
6. The Theta Function	41
7. Proof of the Reciprocity Formula	43
§2.* Theta Functions for Quadratic Forms	44
1. Simple Gaussian Sums	44
2. The Quadratic Reciprocity Law and the Sign of Gaussian Sums	46
3. The Theta Function of a Definite Quadratic Form	48
Chapter II: Ideals and Divisors	
§1. Ideals	53
1. Integral Dependence	53
2. The Finiteness of the Principal Order	54
3. Kronecker Divisors	56
4. Ideals	58
5. Proof of the Principal Theorem	60
6.* Extension of Divisibility Theory	61
§2. Local Rings	63
1. Basic concepts	63
2. Local Rings in Algebraic Extensions	64
3. Local Rings in Algebraic Number and Function Fields	66
4. The Component Decomposition of Ideals	67
§3. Ideals in Different Fields; the Norm	70
1. Extension of an Ideal	70
2. The Norm	70
3. The Prime Ideals	72
§4. The Complement, Different, and Discriminant	73
1. The Complement	73
2. Different and Discriminant	75
3. The Dedekind Discriminant Theorem	77
§5. Divisors	79
1. The Rational Function Field	79
2.* Projective Invariance	81

3. Divisors in Algebraic Number and Function Fields	82
4. The Behavior of Divisors under Field Extensions	84
5. The Prime Divisors	86
6. Divisors and Linear Divisors	88
7. The Linear Degree	89
§6.* Decomposition of Prime Ideals in Galois Extensions	90
1. The Decomposition Group and Inertia Group	91
2. The Ramification Groups	94
3. The Discriminant	96
Appendix to Chapter II:* Topics from the Theory of Algebraic Number Fields	
§1. The Finiteness Theorems	98
1. The Finiteness of the Ideal Class Number	98
2. The Discriminant	100
3. The Dirichlet Unit Theorem	101
4. The Regulator	104
§2. Quadratic Number Fields and Cyclotomic Fields	104
1. Quadratic Number Fields	104
2. Special Cyclotomic Fields	106
Chapter III: Algebraic Functions and Differentials	
§1. Power Series Expansions of Algebraic Functions	110
1. The Field of Power Series	110
2. Divisibility, Rearranging of Power Series	112
3. Inversion of a Power Series	113
4. Algebraic Functions; Regular Places	115
5. Continuation; Critical Places	116
6. Puiseux's Theorem	118
§2. Algebraic Function Fields	120
1. Divisors in Rational Function Fields	120
2. Divisors in Algebraic Function Fields	121
3. Decomposition of Rational Divisors	124
4. The Principal Orders	125
5. Divisors and Linear Divisors	128
6. The Invariance of the Concept of Divisors	131
7.* Extension to More General Constant Fields	131
§3. The Riemann-Roch Theorem	132
1. Dimension of a Divisor Class	132
2. The Riemann-Roch Theorem	133

3. Questions of Invariance	135
4. Extension of the Constant Field	135
5. The Fields of Genus 0	139
6. Lüroth's Theorem	140
7. Further Proofs and Generalizations of the Riemann-Roch Theorem	141
References	142
§4. Differentials	143
1. Differential Quotients	143
2. The Differential Calculus with Characteristic p	144
3. The Concept of the Differential	147
4.* Continuation; Separable and Inseparable Prime Divisors	149
5. Cartier's Operator	150
6. Residues of Differentials	151
7. The Residue Theorem	153
8.* The Differential Class	156
§5. Differentials and Principal Part Systems	158
1. Differentials of Higher Degrees	158
2. Principal Part Systems	159
3. The Scalar Product	160
4. The Relationship to Integral Calculus	163
5.* The Diagonal	165
6.* The Analog of the Green Function	167
Notes	171
§6. Reduction of a Function Field with Respect to a Prime Ideal of the Constant Field	171
1. The Irreducibility Theorem	171
2. Regular Prime Ideals	174
3. Behavior of Ideals under Residue Formation	177
4. Behavior of Divisors under Residue Formation	178
5. Continuation; Behavior of Differentials under Residue Formation	181
6. Behavior of the Field under Residue Formation and Extension	183
Notes	183
References	184
Chapter IV: Algebraic Functions over the Complex Number Field	
§1. Riemann Surfaces	185
1. The Riemann Surface of an Algebraic Function	185
2. The Riemann Surface as Complex Manifold	186
3. The Riemann Surface as Topological Manifold	188
§2. Fields of Elliptic Functions	190
1. Introduction	190
2. The Addition Theorem	191
3. Automorphisms	193

4. The Integral of the First Kind	196
5. The Addition Theorem and the Abel Theorem	197
6. The Weierstrass Normal Form	200
7. Elementary Elliptic Functions	202
Notes	203
References	204
§3. The Group of Divisor Classes of Degree 0	204
1. The Riemann Period Matrix	204
2. A Hermitian Metric for Differentials of the First Kind	206
3. Abelian Integrals of the Third Kind	208
4. Abel's Theorem	209
5. The Jacobian Variety	211
Notes	213
References	214
§4. Modular Functions	214
1. The Modular Surface	214
2. Covering Spaces of the Modular Surface	215
3. Congruence Subgroups	217
4. Modular Forms	219
5. The Field of Modular Functions	221
6. Modular Forms and Differentials	223
7. Fourier Expansions of Eisenstein Series	225
8. Theta Functions	229
References	232
 Chapter V: Correspondences between Fields of Algebraic Functions	
§1. The Correspondences	233
1. Basic Concepts	233
2. Multiplication of Correspondences	236
3. Properties of the Product	239
4. Correspondences of a Field with Itself	241
5. Effect of Correspondences on Divisors	242
6. Prime Correspondences	245
7. Inseparable Extensions	246
8. The Frobenius Automorphism	249
9. Correspondences of a Field of Automorphic Functions with Itself	250
§2. Representations of Correspondences in the Space of Differentials	252
1. Definitions	252
2. The Classical Case	256
3. Continuation; Representations of Rosati-Adjoint Correspondences	258
4. The Trace	259
5. Evaluation of the Trace Formula	263
Notes	265

§3. Modular Functions	266
1. The Modular Correspondences	266
2. Products of Modular Correspondences	271
3. Representations of Modular Correspondences by Differentials	272
4. The Petersson Metric	275
5. Fourier Expansions of Modular Forms	277
6. Ramanujan's Conjecture	279
7. Results for Modular Forms of Odd Dimensions; Notes	281
References	
§4. Castelnuovo's Inequality	281
1. Introduction	282
2. Reduction to the Classical Case	284
3. Extension of the Notion of Correspondence	286
4. The Fixed Points of a Correspondence	289
5. The Connection with §2	291
6. The Trace	293
7. Second Proof of the Principal Theorem: Preparations	295
8. Second Proof of the Principal Theorem: Conclusion	297
9.* Remarks Concerning the Ring of Correspondence Classes	298
Notes	299
References	
§5. Applications in Number Theory	299
1. The Zeta Function of a Field of Functions	299
2. The Functional Equation	301
3. Extension of the Field of Constants	303
4. Riemann's Conjecture	305
5. Modular Functions	307
6. The Eigenvalues of Modular Correspondences	311
7. Modular Functions of the Principal Character	313
Notes	314
References	315
§6. Elliptic Function Fields	315
1. The Ring of Correspondence Classes	316
2. Complex Multiplication	318
AUTHOR INDEX	321
SUBJECT INDEX	322