

# Contents

*Preface*

xi

*Acknowledgments*

xiii

## **Chapter 1 Main Problem and Preliminary Notions**

1.1	Introduction	1
1.2	Main Problem	1
1.3	Topological Manifolds	2
1.4	The Category of Polyhedra and Piecewise Linear Maps	4
1.5	Method of Attacking the Main Problem	8
1.6	Piecewise Linear Manifolds and Piecewise Linear Tools	9
1.7	Local Flatness, (Pinched) Collars, and (Pinched) Bicollars	33
1.8	Cellular Sets and Applications	44

## **Chapter 2 Wild Embeddings, Knotted Embeddings, and Related Topics**

2.1	Introductory Definitions	51
2.2	The Group of a Knot and Knotted Codimension Two Spheres	51
2.3	Local Homotopy Groups, Wild Codimension Two Cells and Spheres, and Tame Nonlocally Flat Codimension Two Cells and Spheres	56
2.4	Wild 1-Cells, 1-Spheres, 2-Cells, and 2-Spheres in $S^3$	60
2.5	$E^n$ Modulo an Arc Crossed with $E^1$ Is $E^{n+1}$	74
2.6	Everywhere Wild Cells and Spheres in $E^{n \geq 3}$ of All Codimensions	84
2.7	Some Wild Polyhedra in Low Codimensions	91

vii

### Chapter 3 Flattening, Unknotting, and Taming Special Embeddings

3.1	Introduction	98
3.2	Almost Polyhedral Arcs Are Flat	99
3.3	An $(n - 1)$ -Sphere in $S^n \geq 4$ which Is Locally Flat Modulo a Point Is Flat	100
3.4	Flattening Cells, Half-Strings, and Strings	105
3.5	PL Unknotting Infinite Polyhedra in the Trivial Range	110
3.6	$\epsilon(x)$ -Taming Locally Tame Embeddings of Infinite Polyhedra in the Trivial Range	118
3.7	$\epsilon$ -Taming Polyhedra in the Trivial Range Which Lie in Hyperplanes	132
3.8	$\epsilon(x)$ -Taming Embeddings Which Are Locally Tame Modulo Nice Subsets	134
3.9	Nonlocally Flat Points of a Codimension One Submanifold	136

### Chapter 4 Engulfing and Applications

4.1	Introduction	148
4.2	Stallings' Engulfing	149
4.3	The Generalized Poincaré Theorem	153
4.4	The Hauptvermutung for Open Cells	154
4.5	Flattening Topological Sphere Pairs and Cell Pairs	158
4.6	Zeeman Engulfing	166
4.7	The Penrose, Whitehead, Zeeman Embedding Theorem and Irwin's Embedding Theorem	170
4.8	The Cellularity Criterion in High Dimensions	178
4.9	Locally Nice Codimension One Spheres in $S^n \geq 5$ Are Weakly Flat	183
4.10	Radial Engulfing	185
4.11	The PL Approximation of Stable Homeomorphisms of $E^n$	193
4.12	Topological Engulfing	200
4.13	Topological $H$ -Cobordisms and the Topological Poincaré Theorem	206
4.14	Infinite Engulfing	214

### Chapter 5 Taming and PL Approximating Embeddings

5.1	Introduction	220
5.2	Černavskii's Straightening Technique Applied to Cell Pairs and to Singular Points of Topological Embeddings	221

5.3	Taming Embeddings of PL Manifolds around the Boundary in All Codimensions	241
5.4	PL Approximating Topological Embeddings	249
5.5	$\epsilon$ -Taming Allowable Embeddings of PL Manifolds	259
5.6	Local Contractibility of the Homeomorphism Group of a Manifold and Codimension Zero Taming	270
<b>Appendix</b>	<b>Some Topics for Further Study</b>	294
<b>Bibliography</b>		296
<i>Author Index</i>		309
<i>Subject Index</i>		312