

# Contents

Chapter 1. The Hilbert Manifold of Closed Curves . . . . .	1
1.1 Hilbert Manifolds . . . . .	1
1.2 The Manifold of Closed Curves . . . . .	7
1.3 Riemannian Metric and Energy Integral of the Manifold of Closed Curves . . . . .	15
1.4 The Condition ( <i>C</i> ) of Palais and Smale and its Consequences . . . . .	22
Chapter 2. The Morse-Lusternik-Schnirelmann Theory on the Manifold of Closed Curves . . . . .	32
2.1 The Lusternik-Schnirelmann Theory on $AM$ . . . . .	32
2.2 The Space of Unparameterized Closed Curves . . . . .	40
2.3 Closed Geodesics on Spheres . . . . .	46
2.4 Morse Theory on $AM$ . . . . .	55
2.5 The Morse Complex . . . . .	65
Chapter 3. The Geodesic Flow . . . . .	77
3.1 Hamiltonian Systems . . . . .	77
3.2 The Index Theorem for Closed Geodesics . . . . .	86
3.3 Properties of the Poincaré Map . . . . .	100
3.3 Appendix. The Birkhoff-Lewis Fixed Point Theorem. <i>By J. Moser</i> . . . . .	115
Chapter 4. On the Existence of Many Closed Geodesics . . . . .	122
4.1 Critical Points in $PM$ and the Theorem of Fet . . . . .	122
4.2 The Theorem of Gromoll-Meyer . . . . .	132
4.3 The Existence of Infinitely Many Closed Geodesics . . . . .	142
4.3 Appendix. The Minimal Model for the Rational Homotopy Type of $AM$ . <i>By J. Sacks</i> . . . . .	156
4.4 Some Generic Existence Theorems . . . . .	161
Chapter 5. Miscellaneous Results . . . . .	167
5.1 The Theorem of the Three Closed Geodesics . . . . .	167
5.2 Some Special Manifolds of Elliptic Type . . . . .	177
5.3 Geodesics on Manifolds of Hyperbolic and Parabolic Type . . . . .	188
Appendix. The Theorem of Lusternik and Schnirelmann . . . . .	203
A.1 The Space $PM$ and the Theorem of Lusternik and Fet . . . . .	204
A.2 Closed Curves without Self-intersections on the 2-sphere . . . . .	208
A.3 The Theorem of Lusternik and Schnirelmann . . . . .	210
Bibliography . . . . .	219
Index . . . . .	225