

## CONTENTS

Preface . . . . .	v
INTRODUCTION . . . . .	1
Part I. ARITHMETICAL SEMIGROUPS AND ALGEBRAIC ENUMERATION PROBLEMS . . . . .	9
CHAPTER 1. ARITHMETICAL SEMIGROUPS . . . . .	11
§1. Integral domains and arithmetical semigroups . . . . .	11
§2. Categories satisfying theorems of the Krull–Schmidt type . . . . .	15
CHAPTER 2. ARITHMETICAL FUNCTIONS . . . . .	23
§1. The Dirichlet algebra of an arithmetical semigroup . . . . .	23
§2. Infinite sums and products . . . . .	26
§3. Double series and products . . . . .	30
§4. Types of arithmetical functions . . . . .	33
§5. The zeta and Möbius functions . . . . .	36
§6. Further natural arithmetical functions . . . . .	39
§7. $\zeta$ -formulae . . . . .	46
CHAPTER 3. ENUMERATION PROBLEMS . . . . .	54
§1. A special algebra homomorphism . . . . .	54
§2. Enumeration and zeta functions in special cases . . . . .	59
§3. Special functions and additive arithmetical semigroups . . . . .	68
Part II. ARITHMETICAL SEMIGROUPS WITH ANALYTICAL PROPERTIES OF CLASSICAL TYPE . . . . .	73
CHAPTER 4. SEMIGROUPS SATISFYING AXIOM A . . . . .	75
§1. The basic axiom . . . . .	75
§2. Analytical properties of the zeta function . . . . .	79

§3. Average values of arithmetical functions . . . . .	90
§4. Approximate average values of special arithmetical functions . . . . .	100
§5. Asymptotic formulae with error estimates . . . . .	104
<b>CHAPTER 5. ASYMPTOTIC ENUMERATION, AND FURTHER “STATISTICAL”</b>	
<b>PROPERTIES OF ARITHMETICAL FUNCTIONS</b> . . . . .	116
§1. Asymptotic enumeration in certain categories . . . . .	116
§2. Maximum orders of magnitude . . . . .	129
§3. Distribution functions of prime-independent functions . . . . .	143
<b>CHAPTER 6. THE ABSTRACT PRIME NUMBER THEOREM</b> . . . . .	
§1. The fundamental theorem . . . . .	154
§2. Asymptotic properties of prime-divisor functions . . . . .	162
§3. Maximum and minimum orders of magnitude of certain functions . . . . .	171
§4. The “law of large numbers” for certain functions . . . . .	177
<b>CHAPTER 7. FOURIER ANALYSIS OF ARITHMETICAL FUNCTIONS</b> . . . . .	
§1. Algebraic and topological theory of Ramanujan sums . . . . .	185
§2. Fourier theory of even functions . . . . .	193
§3. Fourier theory of almost even functions . . . . .	200
§4. A wider type of almost evenness, and pointwise convergence of Ramanujan expansions . . . . .	206
§5. Arithmetical functions over $G_Z$ . . . . .	213
<b>Part III. ANALYTICAL PROPERTIES OF OTHER ARITHMETICAL SYSTEMS</b> . . . . .	
	217
<b>CHAPTER 8. ADDITIVE ARITHMETICAL SEMIGROUPS</b> . . . . .	
§1. Axiom C . . . . .	219
§2. Analytical properties of the zeta function . . . . .	224
§3. The additive abstract prime number theorem . . . . .	232
§4. Further additive prime number theorems . . . . .	241
§5. Asymptotic average values and densities . . . . .	245
<b>CHAPTER 9. ARITHMETICAL FORMATIONS</b> . . . . .	
§1. Natural examples . . . . .	250
§2. Characters and formations . . . . .	251
§3. The $L$ -series of a formation . . . . .	256
	261

§4. Axiom A* . . . . .	264
§5. Analytical properties of $L$ -series . . . . .	270
§6. Average values of arithmetical functions over a class . . . . .	272
§7. Abstract prime number theorem for formations . . . . .	278
<b>APPENDIX 1. SOME UNSOLVED QUESTIONS . . . . .</b>	<b>287</b>
<b>APPENDIX 2. VALUES OF <math>p(n)</math> AND <math>s(n)</math> . . . . .</b>	<b>293</b>
<b>LIST OF SPECIAL SYMBOLS . . . . .</b>	<b>294</b>
<b>BIBLIOGRAPHY . . . . .</b>	<b>297</b>
<b>INDEX . . . . .</b>	<b>319</b>