CONTENTS

PART FOUR

HOMOLOGICAL MANIFOLDS. THE DUALITY THEOREMS. COHOMOLOGY GROUPS OF COMPACTA

Chapter

XIII. Homological Manifolds (h-Manifolds)

1. Definition and Simplest Properties 4
   1.1. Definition of h-manifolds 4
   1.2. Elementary properties of h-manifolds 4
   1.3. The case $n \leq 3$ 6
   1.4. Barycentric stars and h-manifolds 8
2. The Barycentric Complex of a Combinatorial h-Manifold 9
   2.1. Notation; preliminary basic facts 9
   2.2. The complex $\mathfrak{K}^*$ 10
3. The Intersection Number, the Isomorphism D^q and the Poincaré Duality 11
   3.1. The intersection number $(t_i^p \times {}^*t_i^q)$ 11
   3.2. The intersection number and the incidence numbers 14
   3.3. The isomorphism D^q and the Poincaré duality 17
   3.4. The intersection number $(x^p \times {}^*x^q)$ 21
   3.5. Veblen's theorem 23
4. The Combinatorial Form of the Alexander Duality 24
   4.1. Formulation of the theorem 24
   4.2. Generalization of Theorem 4.10 26
   4.3. The cases $p = 0$ and $p = n - 1$; prefatory remarks to the proof of Theorem 4.2 29
   4.4. The isomorphism ΔD^q 30
   4.5. Definition and simplest properties of the linking number $\mathfrak{v}(u^p, u^{q-1})$ 34
   4.6. Theorem on linked systems of cycles 37

XIV. Cohomology Groups of Compacta and the Alexander-Pontryagin Duality

1. Cohomology Groups of Bicompacta 41
   1.1 Spectra and their cohomology groups 41
   1.2. The homomorphism $\bar{S}^\alpha$ 44
   1.3. The cohomology groups of bicompacta 44
   1.4. The group ∇^{00} 45
2. Cofinal Suborders of Spectra. The Case of Compacta 46
   2.1. Cofinal suborders of spectra 46
   2.2. The case of Φ a compactum 47
   2.3. The case of Φ a polyhedron 48

Chapter

3. The Homology Groups of Open Subsets of S^n. Formulation of the Alexander-Pontryagin Duality and Its Fundamental Consequences. Finite Compacta 50
3.1. The group $\Delta^r(\Gamma)$, where Γ is an open subset of S^n 50
3.2. The barycentric subdivision of a chain of Γ 50
3.3. The topological invariance of the groups $\Delta^r(\Gamma)$ 51
3.4. The Alexander-Pontryagin duality and its fundamental consequences 52
3.5. Special case: Φ is finite 54
4. The Complexes K_α, $K_{0\alpha}$, $K_\alpha{}^*$, $K_{0\alpha}{}^*$, $Q_{\alpha 1}$; the Sets G_α and Γ_α 56
4.1. Definitions and notation 56
4.2. The continuous mapping C of G_α onto $\| Q_{\alpha 1} \|$ 59
4.3. Some properties of the group $Z^r(\Gamma)$ 59
5. A Combinatorial Lemma 60
5.1. Introductory remarks and formulation of the lemma .. 60
5.2. The fundamental identity 61
5.3. The completion of the proof of Lemma 5.1 64
6. The Spectrum $\mathfrak{K}_0$ of the Set Φ 66
6.1. The coverings Ω_α and ω_α 66
6.2. The spectrum $\mathfrak{K}_0$ of the compactum Φ 67
6.3. The projections $S_\alpha{}^\beta$ 68
7. Further Auxiliary Propositions 68
7.1. Commutativity of the operators $\bar{S}_\beta{}^\alpha$ and ∇E 68
7.2. Corollary to (5.1) and (7.1) 69
8. Proof of the Alexander-Pontryagin Duality 70
8.1. The homomorphism ΔD^q of $\nabla^p(\Phi)$ into $\Delta^{q-1}(\Gamma)$ induced by the homomorphisms $\Delta D_\alpha{}^q$ 70
8.2. The mapping ΔD^q is an isomorphism 71
8.3. The isomorphism ΔD^q is onto 71

XV. Linking. The Little Alexander Duality

0. Introductory Remarks 73
1. The Intersection Number and the Linking Number 73
1.1. The intersection number of two chains x^p and y^q of R^n. 73
1.2. Computation formulas for intersection numbers 74
1.3. Corollaries of Theorem 1.2. Generalization of the definition of intersection number 77
1.4. The linking number 80
1.5. Computation formulas for the linking numbers 82
1.6. The case $z^p \in Z_\Delta{}^p(K_0)$, $*z^{q-1} \in Z_\Delta{}^{q-1}(K^*)$ 83
2. Linking of Proper Cycles 85

Chapter

2.1. Definition of linking numbers for proper cycles..... 85
2.2. Deformation theorem for singular cycles..... 87
3. The Little Alexander Duality..... 88
3.1. Introductory remarks and formulation of the theorem. 88
3.2. Proof of Theorem (3.1)..... 89
3.3. The little Pontryagin duality..... 91
3.4. Closed $(n-1)$-dimensional pseudomanifolds in S^n $(n \geq 2)$. The Jordan-Brouwer theorem..... 93
3.5. Remarks on linking of pseudomanifolds..... 95

PART FIVE

INTRODUCTION TO THE THEORY OF CONTINUOUS MAPPINGS OF POLYHEDRA

XVI. The Brouwer Theory of Continuous Mappings in R^n and S^n
1. Index of a Point Relative to an $(n-1)$-Cycle in R^n..... 100
1.1. The combinatorial case..... 100
1.2. The index of a point $o \in R^n$ relative to a proper cycle z^{n-1}..... 101
1.3. The Poincaré-Bohl theorem and Rouché's theorem... 103
1.4. Homologies in $R^n \setminus o$..... 103
1.5. The interpretation of the index of a point relative to a cycle as the degree of the central projection of the cycle onto a sphere..... 104
2. The Existence of Roots..... 105
2.1. Existence theorem for roots..... 105
3. The Local Degree of a Mapping of an n-Chain Into R^n.. 106
3.1. Definition and fundamental properties of the local degree..... 107
3.2. Simplicial approximations..... 107
3.3. Mappings into S^n..... 109
4. Topological Mappings..... 109
4.1. Theorem 4.1..... 109
4.2. Invariance theorems..... 110
5. Vector Fields and Continuous Mappings..... 111
5.1. The connection between vector fields and mappings... 111
5.2. The index of an isolated zero of a vector field..... 113
5.3. Vector fields on a solid sphere..... 114
5.4. Brouwer's fixed point theorem for an n-cell..... 115
5.5. Vector fields on spheres of even dimension..... 116

Chapter

5.6. Reflections of spheres; another fixed point theorem ... 117
5.7. Exercises ... 118
6. Classification of Continuous Mappings of an n-Sphere Into an n-Sphere ... 119
6.1. Hopf's theorem; introductory remarks ... 119
6.2. Mappings of degree zero ... 122
6.3. Reduction of Hopf's theorem to an extension theorem. 124
6.4. Proof of the extension theorem ... 125
XVII. Fixed Points of Continuous Mappings of Polyhedra
1. The Fixed Point Theorem ... 128
1.1. Fixed simplexes ... 128
1.2. Hopf's formula ... 129
1.3. The Lefschetz number of a continuous mapping of a polyhedron into itself ... 131
1.4. Examples ... 132
1.5. Some remarks on the Lefschetz number ... 133
2. The Index of a Fixed Point ... 134
2.1. Definition of the index ... 134
2.2. Some properties of the index ... 135
2.3. Normal fixed points. Topological invariance of the index of a fixed point ... 136
2.4. Fixed points of affine mappings ... 138
3. The Algebraic Number of Fixed Points of a Continuous Mapping of a Polyhedron Into Itself ... 138
3.1. Definition of regular fixed points. Formulation of the fundamental theorem ... 138
3.2. Generalization of Theorem 3.1 ... 139
3.3. Regular fixed points of simplicial mappings ... 139
3.4. Reduction of Theorem 3.1 to the approximation theorem ... 140
3.5. Proof of the approximation theorem ... 141
3.6. Remarks on the algebraic number of fixed points ... 144
References ... 146
Index ... 147