

LIST OF CONTENTS

Preface	v
List of Notations	vii
Part 1: p-adic and g-adic Numbers, and Their Approximations	1
I. Valuations and pseudo-valuations	3
1. Valuations and pseudo-valuations	4
2. The p-adic valuations of Γ	5
3. A further example	6
4. Valuations and pseudo-valuations derived from given ones	7
5. Bounded sequences, fundamental sequences, and null sequences	9
6. The ring $\{K\}_w$ and the ideal \mathfrak{p}	11
7. The residue class ring K_w	12
8. The completion of a field with respect to a valuation	13
9. The limit notation	13
10. The continuation of $w(a)$ onto K_w	14
11. The elements of K lie dense in K_w	15
12. Fundamental sequences in K_w	16
13. Equivalence of valuations and pseudo-valuations	17
14. The valuations and pseudo-valuations of Γ	18
15. Independent pseudo-valuations	20
16. The decomposition theorem	21
17. Convergent infinite series	24
II. The p-adic, g-adic, and g^* -adic series	26
1. Notation	27
2. The ring I_g and the ideal \mathfrak{s}	29
3. The residue class ring I_g/\mathfrak{s}	30
4. Systems of representatives	31
5. Series for g-adic numbers	32
6. Series for g^* -adic numbers	36
7. Sequences that converge with respect to all valuations of Γ	40
III. A test for algebraic or transcendental numbers	41
1. Notation	42
2. The minimum polynomial of an algebraic number	42
3. An algebraic identity	43
4. Inequalities for algebraic numbers	45
5. A theorem on linear forms	48
6. On a system of both real and p-adic linear forms	50

7. Polynomials $F(x)$ for which $\omega(F(a))$ is small	53
8. A necessary and sufficient condition for transcendency	55
IV. Continued fractions	58
1. The continued fraction algorithm in the real case	58
2. The convergents of the continued fraction for a	59
3. The distinction between rational and irrational numbers	60
4. Inequalities for $ Q_k a - P_k $	62
5. The convergents as best approximations	63
6. The rational approximations of g -adic integers	63
7. The continued fraction algorithm for a g -adic integer	64
8. Two numerical examples	67
9. Final remarks to the g -adic algorithm	69
10. The continued fraction algorithm for g^* -adic numbers	69
Part 2: Rational Approximations of Algebraic Numbers.	
The Problem and Its History	73
V. 1. Introduction	77
2. Linear dependence and independence	77
3. Generalized Wronski determinants	78
4. The case of functions of one variable	79
5. The general case	80
6. An identity	82
7. Majorants for U , V , and W	85
8. The index of a polynomial	87
9. The upper bound $\theta_m(a; H_1, \dots, H_m; r_1, \dots, r_m)$	89
10. An upper bound for $\theta_1(a; r; H)$	90
11. The property Γ_M	90
12. A recursive inequality for θ_m . I	92
13. A recursive inequality for θ_m . II	93
14. Proof of Roth's Lemma	96
VI. The Approximation Polynomial	98
1. The aim	98
2. The powers of an algebraic number	98
3. A lemma by Schneider	99
4. The construction of $A(x_1, \dots, x_m)$. I.	101
5. The construction of $A(x_1, \dots, x_m)$. II.	102
6. The construction of $A(x_1, \dots, x_m)$. III.	104
VII. The First Approximation Theorem	107
1. The properties A_d , B , and C	107
2. The selection of the parameters	109
3. Application of Theorems 1 and 2	112
4. Upper bounds for $ A(1) $	113
5. An upper bound for $ A(1) _g$	116

6. An upper bound for $ D(l) $	117
7. Lower bounds for $ N(l) $	120
8. Conclusion of the proof	122
9. The first form of the First Approximation Theorem	123
10. Polynomials in a field with a valuation	125
11. Two applications of Lemma 1	127
12. The property A'_d	130
13. The second form of the First Approximation Theorem	131
VIII. The Second Approximation Theorem	133
1. The two forms of the theorem	133
2. The Theorem (2, II) implies the Theorem (2, I)	134
3. The Theorem (2, I) implies the Theorem (2, II)	135
4. The Theorem (2, I) implies the Theorem (1, I)	137
5. The integers e_j	140
6. The numbers $g, g', g'', \rho, \sigma, \lambda, \mu$	143
7. The Theorem (1, I) implies the Theorem (2, I)	145
IX. Applications	147
1. The theorems of Roth and Ridout	147
2. The continued fraction of a real algebraic number	150
3. The powers of a rational number	150
4. The equation $P(k) + Q(k) + R(k) = 0$	155
5. The approximation by rational integers	158
6. An example	161
Appendix A. Another proof of a lemma by Schneider	163
Appendix B. A theorem by M. Cugiani	169
Appendix C. The Approximation Theorems over Algebraic Number Fields	181