

HOW THESE NOTES CAME TO BE

[Although the Chapters are not divided into sections, the list for each Chapter gives some indication which topics are treated, and on which pages. The numbers in brackets after the PROBLEMS refer to Problems which, for one reason or another, I felt should be brought to the attention of the reader.]

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