

[Although the Chapters are not divided into sections, except for the major subdivisions of Chapters 10 and 13, the list for each Chapter gives some indication which topics are treated, and on which pages.]

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