

Contents

Chapter 7

Scalar and Vector Ultra-Distributions.

1. Scalar-Valued Functions of Class M_k	1
1.1 The Sequences $\{M_k\}$	1
1.2 The Space $\mathcal{D}_{M_k}(\Omega)$	2
1.3 The Spaces $\mathcal{D}_{M_k}(\mathcal{X})$ and $\mathcal{E}_{M_k}(\Omega)$	5
2. Scalar-Valued Ultra-Distributions of Class M_k ; Generalizations	6
2.1 The Space $\mathcal{D}'_{M_k}(\Omega)$	6
2.2 Non-Symmetric Spaces of Class M_k	7
2.3 Scalar Ultra-Distributions of Beurling-Type	8
3. Spaces of Analytic Functions and of Analytic Functionals	9
3.1 The Spaces $\mathcal{H}(\mathcal{X})$ and $\mathcal{H}'(\mathcal{X})$	9
3.2 The Spaces $\mathcal{H}(I)$ and $\mathcal{H}'(I)$	10
4. Vector-Valued Functions of Class M_k	11
4.1 The Space $\mathcal{D}_{M_k}(\mathcal{S}; F)$	11
4.2 The Spaces $\mathcal{D}_{M_k}(\mathcal{X}; F)$ and $\mathcal{E}_{M_k}(\mathcal{S}; F)$	12
4.3 The Spaces $\mathcal{D}_{\pm, M_k}(\mathcal{S}; F)$	13
4.4 Remarks on the Topological Properties of the Spaces $\mathcal{D}_{M_k}(\mathcal{S}; F)$, $\mathcal{E}_{M_k}(\mathcal{S}; F)$, $\mathcal{D}_{\pm, M_k}(\mathcal{S}; F)$	15
5. Vector-Valued Ultra-Distributions of Class M_k ; Generalizations	16
5.1 Recapitulation on Vector-Valued Distributions	16
5.2 The Space $\mathcal{D}'_{M_k}(\mathcal{S}; F)$	18
5.3 The Space $\mathcal{D}'_{\pm, M_k}(\mathcal{S}; F)$	20
5.4 Vector-Valued Ultra-Distributions of Beurling-Type	21
5.5 The Particular Case: $F =$ Banach Space	22
6. Comments	22

Chapter 8

Elliptic Boundary Value Problems in Spaces of Distributions and Ultra-Distributions.

1. Regularity of Solutions of Elliptic Boundary Value Problems in Spaces of Analytic Functions and of Class M_k ; Statement of the Problems and Results	25
1.1 Recapitulation on Elliptic Boundary Value Problems	25
1.2 Statement of the M_k -Regularity Results	26
1.3 Reduction of the Problem to the Case of the Half-Ball	29

2.	The Theorem on "Elliptic Iterates": Proof	30
2.1	Some Lemmas	30
2.2	The Preliminary Estimate	32
2.3	Bounds for the Tangential Derivatives	35
2.4	Bounds for the Normal Derivatives	48
2.5	Proof of Theorem 1.3	54
2.6	Complements and Remarks	55
3.	Application of Transposition; Existence of Solutions in the Space $\mathcal{D}'(\Omega)$ of Distributions	58
3.1	Generalities	58
3.2	Choice of the Form L ; the Space $\mathcal{E}(\Omega)$ and its Dual	61
3.3	Final Choice of the Form L ; the Space Y	64
3.4	Density Theorem	65
3.5	Trace Theorem and Green's Formula in Y	66
3.6	The Existence of Solutions in the Space Y	70
3.7	Continuity of Traces on Surfaces Neighbouring Γ	70
4.	Existence of Solutions in the Space $\mathcal{D}'_{M_k}(\Omega)$ of Ultra-Distributions	74
4.1	Generalities	74
4.2	The Space $\mathcal{E}_{M_k}(\Omega)$ and its Dual	75
4.3	The Space Y_{M_k} and the Existence of Solutions in Y_{M_k}	79
4.4	Application to the Regularity in the Interior of Ultra-Distribution Solutions of the Equation $Au = f$	82
5.	Comments	83
6.	Problems	85

Chapter 9

Evolution Equations in Spaces of Distributions and Ultra-Distributions.

1.	Regularity Results. Equations of the First Order in t	87
1.1	Orientation and Notation	87
1.2	Regularity in the Spaces \mathcal{D}_+	88
1.3	Regularity in the Spaces \mathcal{D}_{+,M_k}	91
1.4	Regularity in Beurling Spaces	94
1.5	First Applications	95
2.	Equations of the Second Order in t	98
2.1	Statement of the Main Results	98
2.2	Proof of Theorem 2.1	99
2.3	Proof of Theorem 2.2	100
3.	Singular Equations of the Second Order in t	107
3.1	Statement of the Main Results	107
3.2	Proof of Theorem 3.1	107
4.	Schroedinger-Type Equations	111
4.1	Statement of the Main Results	111
4.2	Proof of Theorem 4.1	111
4.3	Proof of Theorem 4.2	113
5.	Stability Results in M_k -Classes	114
5.1	Parabolic Regularization	114

5.2	Approximation by Systems of Cauchy-Kowaleska Type (I)	117
5.3	Approximation by Systems of Cauchy-Kowaleska Type (II)	121
6.	Transposition	125
6.1	Orientation	125
6.2	The Parabolic Case	125
6.3	The Second Order in t Case and the Schroedinger Case	128
7.	Semi-Groups	129
7.1	Orientation	129
7.2	The Space of Vectors of Class M_k	129
7.3	The Semi-Group G in the Spaces $D(A^\infty; M_k)$. Applications	135
7.4	The Transposed Settings. Applications	143
7.5	Another M_k -Regularity Result	148
8.	M_k -Classes and Laplace Transformation	151
8.1	Orientation-Hypotheses	151
8.2	M_k -Regularity Result	151
8.3	Transposition	153
9.	General Operator Equations	153
9.1	General Results	153
9.2	Application. Periodic Problems	158
9.3	Transposition	160
10.	The Case of a Finite Interval $]0, T[$	161
10.1	Orientation. General Problems	161
10.2	Space Described by $v(0)$ as v Describes X	162
10.3	The Space \mathcal{E}_{M_k}	164
10.4	Choice of L	167
10.5	The Space Y and Trace Theorems	168
10.6	Non-Homogeneous Problems	170
11.	Distribution and Ultra-Distribution Semi-Groups	174
11.1	Distribution Semi-Groups	174
11.2	Ultra-Distribution Semi-Groups	180
12.	A General Local Existence Result	181
12.1	Statement of the Result	181
12.2	Examples	184
13.	Comments	186
14.	Problems	187

Chapter 10

Parabolic Boundary Value Problems in Spaces of Ultra-Distributions.

1.	Regularity in the Interior of Solutions of Parabolic Equations	191
1.1	The Hypocoellipticity of Parabolic Equations	191
1.2	The Regularity in the Interior in Gevrey Spaces	195
2.	The Regularity at the Boundary of Solutions of Parabolic Boundary Value Problems	203
2.1	The Regularity in the Space $\mathcal{D}(\bar{Q})$	203
2.2	The Regularity in Gevrey Spaces	204
3.	Application of Transposition: The Finite Cylinder Case	208
3.1	The Existence of Solutions in the Space $\mathcal{D}'(Q)$: Generalities, the Spaces X and Y	208

3.2	Space Described by $\bar{\mathcal{E}}v$ as v Describes X	212
3.3	Trace and Existence Theorems in the Space Y	216
3.4	The Existence of Solutions in the Spaces $\mathcal{D}'_{s,r}(Q)$ of Gevrey Ultra-Distributions, with $r > 1, s \geq 2m$	221
4.	Application of Transposition: The Infinite Cylinder Case	224
4.1	The Existence of Solutions in the Space $\mathcal{D}'_+(\mathbf{R}; \mathcal{D}'(\Omega))$: The Space X_-	224
4.2	The Existence of Solutions in the Space $\mathcal{D}'_+(\mathbf{R}; \mathcal{D}'(\Omega))$: The Space Y_+ and the Trace and Existence Theorems	229
4.3	The Existence of Solutions in the Spaces $\mathcal{D}'_{+,s}(\mathbf{R}; \mathcal{D}'_r(\Omega))$, with $r > 1, s \geq 2m$	236
4.4	Remarks on the Existence of Solutions and the Trace Theorems in other Spaces of Ultra-Distributions	239
5.	Comments	243
6.	Problems	244

Chapter 11

Evolution Equations of the Second Order in t and of Schroedinger Type.

1.	Equations of the Second Order in t ; Regularity of the Solutions of Boundary Value Problems	246
1.1	The Regularity in the Space $\mathcal{D}(\bar{Q})$	246
1.2	The Regularity in Gevrey Spaces	248
2.	Equations of the Second Order in t ; Application of Transposition and Existence of Solutions in Spaces of Distributions	251
2.1	Generalities	251
2.2	The Space $\mathcal{D}_{-\gamma}([0, T]; \mathcal{D}_{\gamma}(\bar{\Omega}))$ and its Dual	253
2.3	The Spaces X and Y	254
2.4	Study of the Operator $\bar{\mathcal{E}}$	258
2.5	Trace and Existence Theorems in the Space Y	260
2.6	Complements on the Trace Theorems	263
2.7	The Infinite Cylinder Case	265
3.	Equations of the Second Order in t ; Application of Transposition and Existence of Solutions in Spaces of Ultra-Distributions	268
3.1	The Difficulties in the Finite Cylinder Case	268
3.2	The Infinite Cylinder Case for $m > 1$	269
4.	Schroedinger Equations; Complements for Parabolic Equations	273
4.1	Regularity Results for the Schroedinger Equation	273
4.2	The Non-Homogeneous Boundary Value Problems for the Schroedinger Equation	274
4.3	Remarks on Parabolic Equations	276
5.	Comments	277
6.	Problems	277
	Appendix. Calculus of Variations in Gevrey-Type Spaces	279
	Bibliography	290