

# CONTENTS

## PART A: EXTENDABILITY OF A FAMILY OF MEASURES TO A $\sigma$ -ADDITIVE MEASURE (Kolmogorov-Bochner-Minlos Theory)

Introduction . . . . .	3
Chapter 1. Preliminary discussions . . . . .	4
§1. Explanation of the problem . . . . .	4
§2. Tychonov's theorem . . . . .	6
§3. Hopf's theorem . . . . .	9
Chapter 2. Direct product and projective limit . . . . .	14
§4. Measurable spaces, their product and limit . . . . .	14
§5. Extension problems and counter examples . . . . .	17
§6. Extension theorem for direct product . . . . .	22
§7. Extension theorem for projective limit . . . . .	26
§8. Non-countable direct product and projective limit . . . . .	30
§9. Compact regular measurable space . . . . .	35
§10. Borel field . . . . .	38
§11. Baire field . . . . .	42
§12. Product measure . . . . .	46
§13. Suslin set, Luzin set . . . . .	50
§14. Standard measurable space . . . . .	56
Chapter 3. Measures on vector spaces . . . . .	63
§15. Explanation of the problem . . . . .	63
§16. Relation with Bochner's theorem . . . . .	68
§17. Minlos' theorem . . . . .	72
§18. Sazonov topology . . . . .	77
§19. Supplementary results . . . . .	81
§20. Nuclear space . . . . .	85
§21. Heredity . . . . .	89
§22. Dual space . . . . .	95
Appendix. Definition of nuclearity without using Hilbertian semi-norms . . . . .	98
Note . . . . .	106

## PART B: INVARIANCE AND QUASI-INVARIANCE OF MEASURES ON INFINITE DIMENSIONAL SPACES

Introduction . . . . .	111
Chapter 1. Invariant measure on a group . . . . .	113
§1. Measurable group, invariant and quasi-invariant measure . . . . .	113

§2. Haar measure on a locally compact group	118
§3. Haar measure on a thick group	125
§4. Weil topology	133
§5. Case of a vector space	138
<b>Chapter 2. Gaussian measures and related problems</b>	<b>144</b>
§6. Quasi-invariance and ergodicity	144
§7. Absolute continuity of projective limit measures	147
§8. Gaussian measures	151
§9. $E'$ -quasi-invariance and $E'$ -ergodicity	154
§10. Mutual equivalence	160
§11. Rotationally invariant measures	163
§12. Representation of $L^2(\mu)$	167
<b>Chapter 3. The set <math>Y_\mu</math> of all quasi-invariant translations</b>	<b>174</b>
§13. Convolution of measures	174
§14. Linearization of a topology on a vector space	175
§15. Characteristic topology	177
§16. Evaluation of $Y_\mu$ in terms of $\tau_\mu$	181
§17. Some applications	184
§18. Kakutani topology	189
§19. Evaluation of $Y_\mu$ in terms of Kakutani topology	195
<b>Chapter 4. Product measures on <math>\mathbb{R}^\infty</math></b>	<b>202</b>
§20. Product of one-dimensional probability measures	202
§21. Stationary product measures	208
§22. Gaussian measures and stationary products	212
§23. Estimation of $Y_\mu$ and $R_\mu$	216
§24. Non-stationary product measures	219
§25. $(\ell^2)$ -quasi-invariance	222
<b>Chapter 5. <math>\mathbb{R}_0^\infty</math>-invariant measures on <math>\mathbb{R}^\infty</math></b>	<b>227</b>
§26. Infinite dimensional Lebesgue measure	227
§27. $\mathbb{R}_0^\infty$ -ergodicity and mutual equivalence	230
§28. Equivalent probability measure of product type	233
§29. The converse problem of §28	235
§30. Linear transformation of $\mu_\infty$	240
§31. Rotational invariance and ergodicity	244
§32. Invariance under homotheties	247
<b>Note</b>	<b>251</b>
<b>Index</b>	<b>255</b>