

CONTENTS

INTRODUCTION	III
CHAPTER 1 : COXETER GROUPS, HECKE ALGEBRAS AND THEIR REPRESENTATIONS	1
§1.1 Coxeter groups and Weyl groups	1
§1.2 Hecke algebras	5
§1.3 W-graphs	6
§1.4 Kazhdan-Lusztig polynomials	7
§1.5 Cells of a Coxeter group	13
§1.6 The star operations in the sets $\mathcal{D}_L(s,t), \mathcal{D}_R(s,t)$	15
§1.7 Examples of cells	19
CHAPTER 2 : APPLICATIONS OF KAZHDAN-LUSZTIG THEORY	32
§2.1 ϕ -cells of a Coxeter group	33
§2.2 The canonical isomorphism between $\mathbb{Q}[X^{\frac{1}{2}}]W$ and $H_{\mathbb{Q}}[X^{\frac{1}{2}}]$	36
§2.3 Representations of the Weyl group	42
§2.4 Kazhdan-Lusztig conjecture for composition factors of Verma modules	44
§2.5 Classification of primitive ideals in universal enveloping algebras of semisimple Lie algebras	46
§2.6 Modular representation theory of algebraic groups and related finite groups	51
§2.7 Weight multiplicities and Kazhdan-Lusztig polynomials	54
CHAPTER 3 : GEOMETRIC INTERPRETATIONS OF THE KAZHDAN-LUSZTIG POLYNOMIALS	60
§3.1 Complexes of sheaves on an algebraic variety	60
§3.2 The intersection chain complex	62
§3.3 The construction of the intersection chain complex	62
§3.4 The case of Schubert varieties	63
§3.5 The case of the unipotent variety	65
CHAPTER 4 : THE ALGEBRAIC DESCRIPTIONS OF THE AFFINE WEYL GROUPS A_n OF TYPE $\tilde{A}_{n-1}, n > 2$	66
§4.1 Three algebraic descriptions of the affine Weyl group A_n	67
§4.2 The functions $\ell(w), L(w), R(w)$ on the affine Weyl group A_n	68
§4.3 The subsets $\mathcal{D}_L(s_t), \mathcal{D}_R(s_t)$ of the affine Weyl group $A_n, n > 3$	72
§4.4 Some definitions and terminology	72

CHAPTER 5 : THE PARTITION OF n ASSOCIATED WITH AN ELEMENT OF THE AFFINE WEYL GROUP A_n	77
CHAPTER 6 : A GEOMETRICAL DESCRIPTION OF THE AFFINE WEYL GROUP A_n	84
§6.1 The description of A_n as a set of alcoves	84
§6.2 The relation between two descriptions of A_n	91
§6.3 The map $\sigma: A_n \rightarrow \Lambda_n$ defined in geometrical terms	95
CHAPTER 7 : ADMISSIBLE SIGN TYPES OF RANK n	99
§7.1 Admissible sign types and their equivalence relation	99
§7.2 Connected sets of A_n and cells of S	103
§7.3 The cardinality of S	104
CHAPTER 8 : ITERATED STAR OPERATIONS AND INTERCHANGING OPERATIONS ON BLOCKS	114
§8.1 Iterated star operations	115
§8.2 Some results on iterated star operations	117
§8.3 The interchanging operations $\rho_{A_2}^{A_1}$ and $\theta_{A_1}^{A_2}$	118
§8.4 More general interchanging operations	123
CHAPTER 9 : THE SUBSET $\sigma^{-1}(\lambda)$ OF THE AFFINE WEYL GROUP A_n	129
§9.1 Two simple lemmas on iterated star operations	129
§9.2 The subset F of the affine Weyl group A_n	130
§9.3 The subset H_λ of $\sigma^{-1}(\lambda)$	134
§9.4 $\sigma^{-1}(\lambda)$ is a union of RL-equivalence classes	144
CHAPTER 10 : THE SET N_λ OF NORMALIZED ELEMENTS OF $\sigma^{-1}(\lambda)$	146
CHAPTER 11 : THE ORBIT SPACE \tilde{A}_n OF THE AFFINE WEYL GROUP A_n	152
§11.1 Definition of \tilde{A}_n	152
§11.2 The map $\eta: A_n \rightarrow \tilde{A}_n$	153
§11.3 The partition associated with an element of \tilde{A}_n	153
§11.4 The functions $\varrho(\tilde{w})$, $\ell(\tilde{w})$, $R(\tilde{w})$ and star operations in \tilde{A}_n	154
§11.5 Interchanging operations on blocks in \tilde{A}_n	155
§11.6 Totally ordered sets with a distance function	158
§11.7 Deletion operations in \tilde{A}_n	161
§11.8 Commutativity of interchanging operations with deletion	162
§11.9 Commutativity of interchanging operations with the map $\tilde{\eta}$	164
CHAPTER 12 : THE SEQUENCE $\xi(w,k)$ BEGINNING WITH AN ELEMENT OF N_λ	166
§12.1 A description of N_λ	166
§12.2 A sequence $\xi(w,r)$ beginning with an element of H_λ	166
§12.3 The deletion map $d(\lambda,m)$	168

§12.4	The subset $\tilde{H}_{\lambda,k}$ of $\tilde{\sigma}^{-1}(\lambda)$	171
§12.5	The sequence $\xi(\tilde{w},k)$ beginning with $\tilde{w} \in \tilde{N}_\lambda$	178
§12.6	The sequence $\xi(w,k)$ beginning with $w \in N_\lambda$	181
§12.7	Antichains	182
§12.8	The D-function	190
CHAPTER 13 : RAISING OPERATIONS ON LAYERS		202
§13.1	Reflective pairs	202
§13.2	Raising operations on layers	205
§13.3	Proof of Proposition 13.2.3 when $1 < u < \lambda_r$	208
§13.4	Proof of Proposition 13.2.3 when $\lambda_{k+1} < u < \lambda_k$ and $1 < k < r$	215
CHAPTER 14 : THE LEFT AND RIGHT CELLS IN $\sigma^{-1}(\lambda)$		222
§14.1	The map T from N_λ to the set of λ -tabloids	222
§14.2	The set \bar{N}_λ of principal normalized elements	224
§14.3	The subset X_λ of N_λ	228
§14.4	The number of left cells in $\sigma^{-1}(\lambda)$	229
CHAPTER 15 : $\sigma^{-1}(\lambda)$ IS AN RL-EQUIVALENCE CLASS OF A_n		235
CHAPTER 16 : LEFT CELLS ARE CHARACTERIZED BY THE GENERALIZED RIGHT τ -INVARIANT		236
§16.1	Left cells are characterized by the generalized right τ -invariant	236
§16.2	The standard parabolic subgroup P_n	237
CHAPTER 17 : THE TWO-SIDED CELLS OF THE AFFINE WEYL GROUP A_n		243
CHAPTER 18 : SOME PROPERTIES OF CELLS AND OTHER EQUIVALENCE CLASSES OF A_n		247
§18.1	The commutativity between a left star operation and a right star operation	247
§18.2	Connectedness of cells and other equivalence classes of A_n	249
§18.3	The intersection of a left cell with a right cell in A_n	253
CHAPTER 19 : SOME SPECIAL KINDS OF SIGN TYPES		256
§19.1	Coxeter sign types and generalized tabloids	257
§19.2	Three maps $\hat{\sigma}$, f , ϕ and their relations	259
§19.3	Two kinds of actions on the set C	260
§19.4	The map $\hat{f}: A_n \rightarrow \hat{C}$	262
§19.5	Dominant sign types and dominant tabloids	265
§19.6	Special sign types and special tabloids	267

CHAPTER 20 : THE INSERTING ALGORITHM ON THE SET C	273
§20.1 The n -circle and the sets $H_Y(X), L_Y(X)$	273
§20.2 The inserting algorithm on the set C	280
CHAPTER 21 : THE RESTRICTION OF THE MAP \hat{T} ON P_n	285
§21.1 Separated entry sets and the operation $w \xrightarrow{*(i+1, \alpha, m)} y$	285
§21.2 Reformulation of the Robinson-Schensted algorithm	289
§21.3 The image of P_n under the map \hat{T}	291
REFERENCES	296
INDEX OF NOTATION	299
INDEX	303