

CONTENTS

PREFACE	v
ACKNOWLEDGMENTS	vii
I THE BEGINNINGS	1
1. Mesopotamian Mathematics	2
2. The Egyptians	25
II GREEK GEOMETRY	45
1. Thales of Miletus	46
2. The Pythagorean School	48
3. The Athenian School	53
4. Euclid	59
5. Archimedes	65
6. Apollonius	74
7. Greek Cosmology	80
III THE AXIOMATIC METHOD	91
1. Pips and Globes	91
2. Properties of Axiom Systems	105
3. Euclid and the Foundations of Geometry	108
IV HISTORY OF THE PARALLEL POSTULATE	125
1. The Parallel Postulate	125
2. Absolute Geometry	130
3. Absolute Lengths	141

	4. Saccheri	143
	5. Lambert	147
	6. The French Geometers	151
	7. Wolfgang Bolyai	154
	8. Gauss	155
	9. J. Bolyai	160
	10. Lobachevsky	162
V	FUNDAMENTALS OF LOBACHEVSKIAN GEOMETRY	167
	1. Parallelism of Rays	167
	2. Angle of Parallelism	169
	3. Parallelism of Lines—The Angle Criterion	171
	4. Bisector of a Strip	172
	5. Properties of $\Pi(x)$	176
	6. Ideal Points	181
	7. Ideal Triangles	182
	8. More Properties of $\Pi(x)$	185
	9. Divergent Lines	186
	10. Ultra-Ideal Points	188
	11. Sheaves of Lines—Fundamental Curves	190
	12. Limiting Curves	194
	13. Concentric Horocycles	199
VI	THE TRIGONOMETRIC FORMULAS	205
	1. Perpendicular Lines and Planes	205
	2. Parallel Lines and Planes	213
	3. The Limiting Surface	219
	4. Angle of Parallelism Formula	227
	5. Triangle Relations	232
	6. The Three Geometries	239
VII	THE WEIERSTRASS MODEL	247
	1. Preliminaries	248
	2. H^2	253
	3. Distance in H^2	257
	4. Parametric Equation of a Line	262
	5. Angles	265
	6. The Homogeneous Representation	269

CONTENTS	xi
7. Parallels and Horocycles	270
8. Intersections	274
9. Equidistant Curves	275
VIII LOBACHEVSKIAN GEOMETRY AND PHYSICAL SPACE	279
1. Defects and the Parallax of Stars	280
2. The Finite Curved Universe	283
3. Philosophical Objections: Truth or Convenience	288
APPENDIX A: Definitions, Postulates, Propositions of Euclid, Book I	291
APPENDIX B: Hilbert's Postulates	299
APPENDIX C: Hyperbolic Functions	309
APPENDIX D: Vector Geometry and Analysis	313
BIBLIOGRAPHY	321
INDEX	325