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*Note:* The sections marked with \* or \*\* might be omitted in class use if there is a lack of time. (Here the \*\* sections are considered more truly optional.)

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