

CONTENTS

Paragraphs which can be omitted at a first reading are marked with an asterisk. But if you do skip them, then you miss some of the lollipops. The logical dependence of the chapters is given by the Leitfaden.

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