

# C O N T E N T S

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Lecture 9

Topology on  $\mathcal{E}, \mathcal{E}', \mathcal{D}, \mathcal{D}'$ .

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de Rham's theorem.

The first part of de Rham's theorem.

The second part of de Rham's theorem:  
the theorem of closure.

Some consequences of the theorem of  
closure:

(i) Orthogonality relation.

(ii) Poincaré's duality theorem.

Lecture 10

Some applications.

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The third part of de Rham's theorem.

Riemannian manifolds.

Riemannian structure on an arbitrary

$C^\infty$  manifold.

Canonical Euclidean structures in  $T_a^*(V)$

and  $\bigwedge^p T_a^*(V)$ .

Lecture 11

The star operator.

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The star operator on differential forms.

The global scalar product of two  $C^\infty$  forms.

The star operator on currents.

The Riemannian scalar product of a p-current  
and a p-form.

The Laplacian  $\Delta$ .

Lecture 12

The operator  $\Delta$  on functions.

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The elliptic character of  $\Delta$ . Harmonic  
forms.

Compact Riemannian manifolds.

The Hilbert space of square summable forms.

Lecture 13

Intrinsic characterization of  $\mathcal{H}$ .

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Decomposition of  $\mathcal{H}$ .

Cohomology and harmonic forms; Hodge's  
theorem.

Lecture 14

Green's operator  $G$ .

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Decomposition of  $\mathcal{D}$ .

Continuity of  $G$ .

Self-adjointness of  $G$ .

### Lecture 15

Decomposition of  $\mathcal{D}'$ .  
Commutativity of an operator with  
 $\Delta$ ,  $\pi_1$  and  $G$ .

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### Lecture 16

Real vector spaces with a  $J$ -structure

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The operators  $\frac{\partial}{\partial z_j}$  and  $\frac{\partial}{\partial \bar{z}_j}$ .

Holomorphic functions on  $\mathbb{C}^n$ .

Transformation formulæ.

Canonical complex structure on  $\mathbb{R}^{2n}$ .

Complex analytic manifolds.

Some examples of complex analytic manifolds.

### Lecture 17

The operator  $J$ .

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Bigradation for differential forms.

Holomorphic functions and forms.

The operators  $d_z$  and  $d_{\bar{z}}$ .

$z$  and  $\bar{z}$  cohomology.

Intrinsic characterization of holomorphic forms.

Holomorphic forms and  $\bar{z}$  cohomology.

### Lecture 18

The canonical orientation of a complex manifold.

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Currents.

Ellipticity of the system  $\frac{\partial}{\partial z}$ .

Ellipticity of  $d_{\bar{z}}$  on  $\mathcal{D}' \otimes \bar{\mathcal{D}}^k$ .

$J$ -Hermitian forms.

Hermitian manifolds.

Kählerian manifolds.

### Lecture 19

Some more operators.

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Commutativity relations in a Kählerian manifold.

Holomorphic forms on a Kählerian manifold.

### Lecture 20

Proof of the formula  $[\Lambda, d_z] = i \frac{\partial}{\partial \bar{z}}$

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## Lecture 21

Compact manifolds with a Kählerian structure.

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The map  $\Omega : \bigwedge^{r-1, s-1} \longrightarrow \bigwedge^{r, s}$

The space  $H^{(p, 0)}$

Compact Riemann surfaces.

## Lecture 22

The identity between  $d$ ,  $z$  and  $\bar{z}$  cohomologies, de Rham's theorem (Compact Kählerian manifolds).

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de Rham's theorem for  $\bar{z}$  cohomology for an arbitrary complex analytic manifold.

The complex projective space.

Example of a compact complex manifold which is not Kählerian.

## Lecture 23

Cousin's problem.

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## Lecture 24

Cousin's problem on  $C^n$ .

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Cousin's problem on a compact Kählerian manifold; pseudo solution.

Cousin's problem on  $PC^n$ .

Cousin's problem on a compact Riemann surface.

## Lecture 25

Some applications.

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The Riemann-Roch theorem.

## Appendix

Kählerian structure on the complex projective space.

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