

CONTENTS

O.	Introduction	1
0.1	Abstract	1
0.2	The problem and its solution	1
0.3	Survey of the proof, contents of the chapters	8
0.4	More general questions	11
0.5	Acknowledgements	13
I.	Compact presentability and contracting automorphisms	15
1.1	Compact presentability	15
1.2	Contracting automorphisms	17
1.3	Compact presentability and contracting automorphisms	23
II.	Filtrations of Lie algebras and groups	27
2.1	Filtered Lie algebras	27
2.2	Formulae on commutators	30
2.3	Filtered groups	31
2.4	Some results about nilpotent groups	33
2.5	Nilpotent Lie groups over fields of characteristic zero	36
2.6	Nilpotent Lie groups over p -adic fields	44
III.	A necessary condition for compact presentability	49
3.1	Weights	50
3.2	Values of weights	52
3.3	A counterexample	56
3.4	The geometric invariant of Bieri and Strebel	59
IV.	Implications of the necessary condition	61
4.1	The building blocks N^C	62
4.2	The colimits H and M	64
4.3	Commutators in n	68
4.4	The descending central series of M	71

4.5	H changed into a topological group H_U	80
4.6	The kernel of $H_U \rightarrow Q \rtimes N$	83
4.7	The colimit of the Lie algebras \mathfrak{n}^C	87
V.	The second homology	90
5.1	Homology of a group. The Hopf extension	91
5.2	Homology of a Lie algebra	93
5.3	Lie algebra homology versus group homology	96
5.4	A topology on the Hopf extension	102
5.5	$H_2(\mathfrak{n} \mathbb{Q}_p)^\circ \cong H_2(\mathfrak{n} K)^\circ$	107
5.6	The main theorem	111
5.7	Examples	115
VI.	S-arithmetic groups	123
6.1	Kneser's result	124
6.2	The split solvable case	125
6.3	A maximal split solvable subgroup	128
6.4	G split	129
6.5	Proof of theorem 6.4.3	133
6.6	Extending a representation of a Borel subgroup	137
VII.	S-arithmetic solvable groups	146
7.1	Facts about S-arithmetic groups	148
7.2	The commutator subgroup	150
7.3	The Bieri-Strebel invariant	153
7.4	The second homology	158
7.5	G not connected	163
Appendix.	Linear inequalities	169
References		171
List of symbols		175
Index		177