

CONTENTS

page

PREFACE

viii

Chapter I. METRIC SPACES AND NORMED LINEAR SPACES

1

1. DEFINITIONS AND EXAMPLES

1

Metric spaces, normed linear spaces; metrics generated by a norm; co-ordinate, sequence and function spaces; semi-normed linear spaces; Exercises.

2. BALLS AND BOUNDEDNESS

21

Balls and spheres in metric spaces and normed linear spaces, relating norms and balls; boundedness, diameter; distances between sets; Exercises.

Chapter II. LIMIT PROCESSES

36

3. CONVERGENCE AND COMPLETENESS

36

Convergence of sequences, characterisation in finite dimensional normed linear spaces, uniform convergence; equivalent metrics and norms; Cauchy sequences, completeness; convergence of series; Exercises.

4. CLUSTER POINTS AND CLOSURE

66

Cluster points, closed sets; relating closed to complete; closure, density, separability; the boundary of a set; Exercises.

5. APPLICATION: BANACH'S FIXED POINT THEOREM

91

Fixed points, Banach's Fixed Point Theorem.

5.7 Application in real analysis

93

5.8 Application in linear algebra

96

5.9 Application in the theory of differential equations

100

Picard's Theorem

5.10 Application in the theory of integral equations

103

Fredholm integral equations, Volterra integral equations

Exercises.

Chapter III. <u>CONTINUITY</u>	114
6. CONTINUITY IN METRIC SPACES	114
Local continuity, characterisation of continuity by sequences, algebra of continuous mappings; global continuity characterised by inverse images; isometrics, homeomorphisms; uniform continuity; Exercises.	
7. CONTINUOUS LINEAR MAPPINGS	138
Characterisation of continuity of linear mappings, linear mappings on finite dimensional normed linear spaces, continuity of linear functionals; topological isomorphisms, isometric isomorphisms; Exercises.	
Chapter IV. <u>COMPACTNESS</u>	160
8. SEQUENTIAL COMPACTNESS IN METRIC SPACES	161
Properties of compact sets; characterisation in finite dimensional normed linear spaces, Riesz Theorem; application in approximation theory; alternative forms of compactness, total boundedness, ball cover compactness; separability; Exercises.	
9. CONTINUOUS FUNCTIONS ON COMPACT METRIC SPACES	183
Heine's Theorem, Dini's Theorem	
9.9 The structure of the real Banach space $(C[a, b], \ \cdot\ _\infty)$	187
The Weierstrass Approximation Theorem	
9.10 The structure of the Banach space $(C(X), \ \cdot\ _\infty)$ where (X, d) is a compact metric space	194
9.11 Compactness in $(C(X), \ \cdot\ _\infty)$	200
equicontinuity, The Ascoli-Arzelà Theorem, Peano's Theorem	
Exercises.	

Chapter V. <u>THE METRIC TOPOLOGY</u>	213
10. THE TOPOLOGICAL ANALYSIS OF METRIC SPACES	214
Open sets and their properties, base for a topology; equivalent metrics; relation to closed sets; the interior of a set; the characterisation of continuous mappings by inverse images; topological compactness; separability, the normal topological structure; Exercises.	
<u>APPENDICES</u>	235
Appendix 1. The real analysis background	235
Appendix 2. The set theory background	240
Appendix 3. The linear algebra background	246
INDEX TO NOTATION	251
INDEX	253