CONTENTS

	Introduction	1
1	The fundamental theorem of arithmetic	5
1-24	Division algorithm	5
25–42	Greatest common divisor and Euclidean algorithm	11
43–61	Unique factorisation into primes	13
62–66	Infinity of primes	17
67	Mersenne primes	17
	Notes and answers	18
	Historical note	23
2	Modular addition and Euler's ϕ function	24
1-18	Congruence classes and the Chinese remainder theorem	24
19-38	The groups $(\mathbf{Z}_n, +)$ and their generators	29
39–56	Euler's ϕ function	35
57–64	Summing Euler's function over divisors	38
	Notes and answers	40
	Historical note	47
3	Modular multiplication	50
1-20	Fermat's theorem	50
21-25	Wilson's theorem	55
26-33	Linear congruences	55
34-42	Fermat-Euler theorem	56
43-44	Simultaneous linear congruences	57
45-57	Lagrange's theorem for polynomials	58
58-74	Primitive roots	63
75–87	Chevalley's theorem	66
	Notes and answers	69
	Historical note	76
4	Quadratic residues	77
1–29	Quadratic residues and the Legendre symbol	77

Contents

30–43	Gauss' lemma	19
44–65	Law of quadratic reciprocity	82
	Notes and answers	86
	Historical note	93
5	The equation $x^n + y^n = z^n$, for $n = 2, 3, 4$	95
1-18	The equation $x^2 + y^2 = z^2$	95
19-23	The equation $x^4 + y^4 = z^4$	97
24-26	The equation $x^2 + y^2 + z^2 = t^2$	98
27-68	The equation $x^3 + y^3 = z^3$	99
	Notes and answers	105
	Historical note	113
6	Sums of squares	115
1-36	Sums of two squares	115
37-52	Sums of four squares	119
53-54	Sums of three squares	121
	Notes and answers	122
	Historical note	130
7	Partitions	131
1-15	Ferrers' graphs	131
16-35	Generating functions	132
36-47	Euler's theorem	136
	Notes and answers	138
	Historical note	143
8	Quadratic forms	144
1-20	Unimodular transformations	144
21-31	Equivalent quadratic forms	148
32-43	Discriminant	152
44-52	Proper representation	154
53-72	Reduced forms	155
73–77	Automorphs of definite quadratic forms	158
	Notes and answers	160
	Historical note	175
9	Geometry of numbers	176
1-28	Subgroups of a square lattice	176
29-46	Minkowski's theorem in two dimensions	181
47-66	Subgroups of a cubic lattice	186
67-73	Minkowski's theorem in three dimensions	189
74-86	Legendre's theorem on $ax^2 + by^2 + cz^2 = 0$	190
	Notes and answers	193
	Historical note	201
10	Continued fractions	202
1–7	Irrational square roots	202
8-25	Convergence	202

vi

Conter	Contents	
26-53	Purely periodic continued fractions	208
54-71	Pell's equation	211
72–77	Lagrange's theorem on quadratic irrationals	214
78-82	Automorphs of the indefinite form $ax^2 - by^2$	215
	Notes and answers	218
	Historical note	228
11	Approximation of irrationals by rationals	229
1-10	Naive approach	229
11-22	Farey sequences	230
23-33	Hurwitz' theorem	232
34-43	Liouville's theorem	234
	Notes and answers	238
	Historical note	243
	Bibliography	244
	List of definitions and theorems	246
	Index	255