

Contents

Preface	v
Chapter 0. INTRODUCTION	xv
Chapter 1. AXIOMATIC METHOD	1
§ 1. Projective Plane	1
§ 2. Affine Plane	3
§ 3. Models	4
3.1. Simplicity	6
3.2. Independence	6
3.3. Consistency	7
3.4. Completeness	7
3.5. Categorical	7
§ 4. Finite Projective Planes	8
4.1. Exercises	12
§ 5. Latin Squares and Euler Conjecture	13
5.1. Example	20
5.2. Exercises	21
§ 6. Euclidean Geometry	22
6.1. Undefined elements of Euclidean geometry	23
6.2. Euclid's axioms	23
6.3. Criticism	23
6.4. Contents of axiomatic systems to replace Euclid's axioms	25
6.5. Concept of distance	26
6.6. Distance on Euclidean plane	27
6.7. Order on Euclidean plane	28
6.8. Angle on Euclidean plane	28
6.9. Axioms of continuity and completeness	28
6.10 Exercises	30
§ 7. Non-Euclidean Geometry	32
7.1. Three Non-Euclidean Geometries	33
7.2. Exercises	37

Chapter 2. SYNTHETIC PROJECTIVE GEOMETRY	40
§ 8. Terminology and Notation	40
§ 9. The Principle of Duality	40
9.1. Exercises	41
§ 10. Configurations and Fano's Axiom	42
§ 11. Quadrangle and Quadrilateral	44
11.1. Exercises	47
§ 12. Desargues Axiom	48
12.1. Exercises	51
§ 13. Harmonic Sets	51
13.1. Exercises	55
§ 14. Axioms of Order	56
14.1. Exercises	60
§ 15. Perspectivity and Projectivity	61
15.1. Introduction	61
15.2. Definitions and Theorems	61
15.3. Exercises	70
§ 16. Net of Rationality and Continuity Axiom	71
16.1. Definitions and Theorems	71
16.2. Exercises	77
Chapter 3. ANALYTIC PROJECTIVE GEOMETRY	78
§ 17. Algebraic Preliminaries	78
17.1. Definitions and Theorem	78
17.2. Exercises	82
§ 18. Matrices and Determinants	83
18.1. Matrices	83
18.2. Determinants	88
18.3. Examples	92
18.4. Matrices (Revisited)	93
18.5. Example	94
18.6. Exercises	94
18.7. Applications	97
18.8. Examples	98
18.9. Linear Transformation of R^n	100
18.10. Exercises	101

§ 19.	Equivalence Relation	102
	19.1. Examples	103
	19.2. Exercises	104
§ 20.	Analytic Projective Plane	105
	20.1. Exercises	108
§ 21.	Coordinates for P^2	109
	21.1. Exercises	112
§ 22.	Coordinate System for a Line	113
	22.1. Examples	114
§ 23.	Desargues' Axiom	116
§ 24.	Change of Coordinates	117
§ 25.	Non-Singular Linear Transformations of R^2	121
	25.1. Example	122
	25.2. Exercises	123
§ 26.	Cross Ratio	124
§ 27.	Projective Transformations of a Line onto itself	130
	27.1. Exercises	131
§ 28.	Polarities and Conics	132
	28.1. Intersection of a Line and a Conic	134
	28.2. Example	136
	28.3. Exercises	137
Chapter 4.	ERLANGER PROGRAM	140
§ 29.	Group of Transformations	140
	29.1. Example	140
	29.2. Exercises	141
§ 30.	Group of Projective Transformations	142
	30.1. Example	145
	30.2. Exercises	146
§ 31.	Subgroups	147
	31.1. Example	147
	31.2. Exercises	148
§ 32.	Analytic Affine Plane	148

§ 33.	Classification of Conics	154
	33.1. Example	155
	33.2. Exercises	157
§ 34.	Euclidean Plane and the Group of Euclidean Transformations	158
	34.1. Exercises	162
§ 35.	Other Subgeometries of Projective Geometry	162
	35.1. Exercises	165
	35.2. Tree diagram	165
Chapter 5.	TOPOLOGY—A NEW GEOMETRY	166
§ 36.	Topological Spaces	166
	36.1. Examples	167
	36.2. Examples	169
	36.3. Examples	170
	36.4. Exercises	171
§ 37.	Continuous Transformations	171
	37.1. Examples	172
	37.2. Exercises	174
§ 38.	Homeomorphisms	174
	38.1. Example	175
	38.2. Exercises	175
§ 39.	Product Topology	175
	39.1. Examples	176
	39.2. Examples	178
	39.3. Exercises	179
§ 40.	Identification Topology	180
	40.1. Examples	180
	40.2. Exercises	182
§ 41.	Topology of the Projective Plane	182
Chapter 6.	HIGHER DIMENSIONAL SPACES	187
§ 42.	Vector Spaces	187
	42.1. Examples of Vector Spaces	188
	42.2. Complex Numbers	190
	42.3. Exercises	191

42.4. Examples	194
42.5. Exercises	195
§ 43. <i>n</i> -Dimensional Euclidean Space	196
43.1. Introduction	196
43.2. The Space \mathbb{R}^3	198
43.3. Exercises	201
§ 44. Euclidean Transformations	202
44.1. Examples	205
44.2. Exercises	206
§ 45. <i>n</i> -Dimensional Affine Space	207
45.1. Introduction	207
45.2. Example	211
45.3. Exercises	211
§ 46. <i>n</i> -Dimensional Projective Space	212
46.1. Example	213
46.2. Exercises	215
§ 47. Topology of Higher Dimensional Spaces	215
REFERENCES	218
APPENDIX	220
1. Definitions, Postulates	220
2. Hilbert's Axioms	225
3. Birkhoff's Postulates	232
4. S M S G Postulate	233
ANSWERS AND HINTS	237
INDEX	255