

Contents

Preface	9
Notation	11
Chapter 1 Preliminaries	
1.1 Set Theory	15
1.2 Topological Ideas	17
1.3 Sequences and Limits	18
1.4 Functions and Mappings	21
1.5 Cardinal Numbers and Countability	22
1.6 Further Properties of Open Sets	23
1.7 Cantor-like Sets	23
Chapter 2 Measure on the Real Line	
2.1 Lebesgue Outer Measure	27
2.2 Measurable Sets	30
2.3 Regularity	35
2.4 Measurable Functions	37
2.5 Borel and Lebesgue Measurability	42
2.6 Hausdorff Measures on the Real Line	45
Chapter 3 Integration of Functions of a Real Variable	
3.1 Integration of Non-negative Functions	54
3.2 The General Integral	60
3.3 Integration of Series	68
3.4 Riemann and Lebesgue Integrals	71
Chapter 4 Differentiation	
4.1 The Four Derivates	77

4.2	Continuous Non-differentiable Functions	79
4.3	Functions of Bounded Variation	81
4.4	Lebesgue's Differentiation Theorem	84
4.5	Differentiation and Integration	87
4.6	The Lebesgue Set	90
Chapter 5 Abstract Measure Spaces		
5.1	Measures and Outer Measures	93
5.2	Extension of a Measure	95
5.3	Uniqueness of the Extension	99
5.4	Completion of a Measure	100
5.5	Measure Spaces	102
5.6	Integration with respect to a Measure	105
Chapter 6 Inequalities and the L^p Spaces		
6.1	The L^p Spaces	109
6.2	Convex Functions	111
6.3	Jensen's Inequality	113
6.4	The Inequalities of Hölder and Minkowski	115
6.5	Completeness of $L^p(\mu)$	118
Chapter 7 Convergence		
7.1	Convergence in Measure	121
7.2	Almost Uniform Convergence	125
7.3	Convergence Diagrams	128
7.4	Counterexamples	131
Chapter 8 Signed Measures and their Derivatives		
8.1	Signed Measures and the Hahn Decomposition	133
8.2	The Jordan Decomposition	137
8.3	The Radon-Nikodym Theorem	139
8.4	Some Applications of the Radon-Nikodym Theorem	142
8.5	Bounded Linear Functionals on L^p	147
Chapter 9 Lebesgue-Stieltjes Integration		
9.1	Lebesgue-Stieltjes Measure	153
9.2	Applications to Hausdorff Measures	157
9.3	Absolutely Continuous Functions	160
9.4	Integration by Parts	163
9.5	Change of Variable	167
9.6	Riesz Representation Theorem for $C(I)$	172

Chapter 10 Measure and Integration in a Product Space	
10.1 Measurability in a Product Space	176
10.2 The Product Measure and Fubini's Theorem	179
10.3 Lebesgue Measure in Euclidean Space	185
10.4 Laplace and Fourier Transforms	189
Hints and Answers to Exercises	
Chapter 1	197
Chapter 2	198
Chapter 3	204
Chapter 4	209
Chapter 5	211
Chapter 6	215
Chapter 7	220
Chapter 8	223
Chapter 9	227
Chapter 10	230
References	236
Index	237