Contents

PART I CALCULUS OF ONE VARIABLE 1

Chapter 1 The Number System *3*

1.1 The Peano Axioms 3
1.2 Rational Numbers and Arithmetic 6
1.3 The Real Numbers: Completeness 9
1.4 Geometry and the Number System 13
1.5 Bounded Sets 15
1.6 Some Points of Logic 18
1.7 Absolute Value 19

Chapter 2 Functions, Sequences, and Limits *25*

2.1 Mappings, Functions, and Sequences 25
2.2 Limits 30
2.3 Operations with Limits (Sequences) 37
2.4 Limits of Functions 44
2.5 Operations with Limits (Functions) 49
2.6 Monotone Sequences 53
2.7 Monotone Functions 56

Chapter 3 Continuity and More Limits *61*

3.1 Continuity. Uniform Continuity 61
3.2 Operations with Continuous Functions 65
3.3 The Intermediate-Value Property 67
3.4 Inverse Functions 68
3.5 Cluster Points. Accumulation Points 74
3.6 The Cauchy Criterion 79
3.7 Limit Superior and Limit Inferior 84
3.8 Deeper Properties of Continuous Functions 89

Chapter 4 Differentiation *95*

4.1 The Derivative. Chain Rule 95
4.2 The Mean-Value Theorem 101
4.3 The Cauchy Mean-Value Theorem 111
4.4 L'Hospital's Rule 113
4.5 Taylor's Formula with Remainder 120
4.6 Extreme Values 126

Chapter 5 Integration *131*

5.1 Introduction 131
5.2 Preliminary Lemmas 133
5.3 The Riemann Integral 140
5.4 Properties of the Definite Integral 151
5.5 The Fundamental Theorem of Calculus 156
5.6 Further Properties of Integrals 160
5.7 Integrals of Discontinuous Functions 166

Chapter 6 The Elementary Transcendental Functions *175*

6.1 The Logarithm 175
6.2 The Exponential Function 179
6.3 The Circular Functions 183

PART II VECTOR CALCULUS 197

Chapter 7 Vectors and Curves *199*

7.1 Introduction and Definitions 199
7.2 Vector Multiplications 206
7.3 The Triple Products 214
7.4 Linear Independence. Bases. Orientation 219
7.5 Vector Analytic Geometry 223
7.6 Vector Spaces of Other Dimensions: E_n 225
7.7 Vector Functions. Curves 230
7.8 Rectifiable Curves and Arc Length 233
7.9 Differentiable Curves 237

Chapter 8 Functions of Several Variables. Limits and Continuity *249*

8.1 A Little Topology: Open and Closed Sets 249
8.2 A Little More Topology: Sequences, Cluster Values, Accumulation Points, Cauchy Criterion 254
8.3 Limits 260
8.4 Vector Functions of a Vector 264
8.5 Operations with Limits 268
8.6 Continuity 269
8.7 Geometrical Picture of a Function 274
8.8 Matrices and Linear Transformation 277

Chapter 9 Differentiable Functions *289*

9.1 Partial Derivatives 289
9.2 Differentiability. Total Differentials 299
9.3 The Derivative 307
9.4 The Gradient. The Del Operator. Directional Derivatives 315
9.5 The Chain Rule 321
9.6 The Mean Value Theorem and Taylor's Theorem for Several Variables 330
9.7 The Divergence and Curl of a Vector Field 333

Chapter 10 The Inversion Theorem *339*

10.1 Transformations. Inverse Transformations 339
10.2 The Inversion Theorem 341
10.3 Implicit Functions 350
10.4 Global Inverses 358
10.5 Curvilinear Coordinates 360
10.6 Extreme Values 368
10.7 Extreme Values Under Constraints 372

Chapter 11 Multiple Integrals *379*

11.1 Integrals Over Rectangles 379
11.2 Properties of the Integral. Classes of Integrable Functions 387
11.3 Iterated Integrals 389
11.4 Integration Over Regions. Area and Volume 394

Chapter 12 Line and Surface Integrals *405*

12.1 Line Integrals. Potentials 405
12.2 Green's Theorem 417
12.3 Surfaces. Area 429
12.4 Surface Integrals. The Divergence Theorem 435
12.5 Stokes' Theorem. Orientable Surfaces 442
12.6 Some Physical Heuristics 449
12.7 Change of Variables in Multiple Integrals 451

PART III THEORY OF CONVERGENCE **461**

Chapter 13 Infinite Series *463*

13.1 Convergence, Absolute and Conditional 463
13.2 Series with Nonnegative Terms: Comparison Tests 468
13.3 Series with Nonnegative Terms: Ratio and Root Tests. Remainders 475
13.4 Series with Variable Signs 480

13.5 More Delicate Tests 483
13.6 Rearrangements 486
13.7 Improvement of Convergence 492

Chapter 14 Sequence and Series of Functions. Uniform Convergence ***503***

14.1 Introduction 503
14.2 Uniform Convergence 504
14.3 Consequences of Uniform Convergence 510
14.4 Abel's and Dirichlet's Tests 521
14.5 A Theorem of Dini 525

Chapter 15 The Taylor Series ***529***

15.1 Power Series. Interval of Convergence 529
15.2 Properties of Power Series 536
15.3 The Taylor and Maclaurin Series 543
15.4 The Arithmetic of Power Series 549
15.5 Substitution and Inversion 558
15.6 Complex Series 560
15.7 Real Analytic Functions 564

Chapter 16 Improper Integrals ***567***

16.1 Improper Integrals. Conditional and Absolute Convergence 567
16.2 Improper Integrals with Nonnegative Integrands 576
16.3 The Cauchy Principal Value 579
16.4 An Alternation Test 581
16.5 Improper Multiple Integrals 584

Chapter 17 Integral Representations of Functions ***591***

17.1 Introduction. Proper Integrals 591
17.2 Uniform Convergence 595
17.3 Consequences of Uniform Convergence 601

Chapter 18 Gamma and Beta Functions. Laplace's Method and Stirling's Formula *621*

18.1 The Gamma Function 621
18.2 The Beta Function 625
18.3 Laplace's Method 629
18.4 Stirling's Formula 635

Chapter 19 Fourier Series *639*

19.1 Introduction 639
19.2 The Class $\mathcal{R}_2$. Approximation in the Mean. Bessel's Inequality 646
19.3 Some Useful Lemmas 650
19.4 Convergence Theorems 654
19.5 Differentiation and Integration. Uniform Convergence 664
19.6 Sine and Cosine Series. Change of Scale 669
19.7 Improvement of Convergence 673
19.8 The Fourier Integral 676
19.9 Function Spaces. Complete Orthonormal Sets 683

Elementary Differentiation and Integration Formulas *691*

Answers, Hints, and Solutions *693*

Index *727*