

Contents

Part 1

- 1 *Early geometry* 1
The contrast between Greek and Babylonian mathematics raised. The limitations of rhetorical algebra. The earliest Greek mathematics, a search for deductive validity. Pythagoras' theorem known to the Babylonians, c. 1700 B.C.; a dissection proof based on the theory of figured numbers. Incommensurable magnitudes shown to exist; a conjectural account of their discovery. Reasons for the retention of geometry discussed. Exercises. Pythagorean triples, Greek and Babylonian methods. Plimpton 322. Theodorus on $\sqrt{3}$ to $\sqrt{17}$
- 2 *Incommensurability* 18
The discovery of the irrationality of $\sqrt{2}$ led to further research into incommensurability, but not to a foundational crisis. Similarity and parallelism. Eudoxus on the nature of ratio. Exercises. The application of areas and 'geometric algebra'. Side and diameter numbers and the Euclidean algorithm
- 3 *Euclidean geometry and the parallel postulate* 29
Geometry as the study of figures in space, and assumptions about space. Euclid's *Elements*. The parallel postulate; is such an assumption necessary? Consequences of the postulate; existence and uniqueness of parallels; equidistant lines. Various arguments to 'prove' the postulate, including those of Proclus, Aganis, and Nasir Eddin al-Tusi. Assumptions equivalent to the parallel postulate. Appendix on solid geometry and trigonometry. Exercises. A geometry on the sphere. Plane and spherical trigonometry

Part 2

- 4 *Saccheri and his Western predecessors* 51
Revival of interest in the West in the 16th and 17th centuries. The work of Saccheri. Three hypotheses. HOA refuted. HAA discussed
- 5 *J. H. Lambert's work* 63
Spherical geometry. The work of J. H. Lambert. Absolute nature of length. Angle sum and area. The imaginary sphere. Exercises

6	<i>Legendre's work</i>	69
	French lack of interest; except for Legendre. His 'refutations' of non-Euclidean geometries. Exercises	
7	<i>Gauss's contribution</i>	74
	Kant. Gauss's work. Directed lines. A new definition of parallel. Corresponding points. The horocycle. Exercises	
8	<i>Trigonometry</i>	82
	Trigonometric and hyperbolic functions. Exercises	
9	<i>The first new geometries</i>	87
	Schweikart's Astral geometry. Taurinus' logarithmic-spherical geometry. Approximate agreement between the new geometries and the old. Appendix. Exercises	
10	<i>The discoveries of Lobachevskii and Bolyai</i>	96
	The Bolyai's struggle. Absolute geometry. The work of Lobachevskii summarized and described. The prism theorem; the horocycle and horosphere; the projection map. Geometry on the horosphere is Euclidean; the fundamental formulae of hyperbolic geometry. Bolyai's work; squaring the circle. Summary. Priorities. Exercises. Appendix on spherical trigonometry, including Bolyai's proof of its absolute nature	
11	<i>Curves and surfaces</i>	117
	Curves. Surfaces; co-ordinates; curvature. Intrinsic and extrinsic viewpoints. Geodesics. Minding's surface. Appendix on degeneracies	
12	<i>Riemann on the foundations of geometry</i>	129
	Riemann's hypotheses. Co-ordinates on surfaces. Intrinsic geometry and curvature; metrical ideas at the basis of geometry	
13	<i>Beltrami's ideas</i>	135
	Beltrami's model. Relative consistency of mathematics; foundational questions. Bolyai-Lobachevskii formulae	
14	<i>New models and old arguments</i>	142
	Klein's model of elliptic geometry. Poincaré's conformal model of hyperbolic geometry. Revisiting the work of Wallis, Saccheri, and Legendre. Exercises	
15	<i>Resumé</i>	155
	Summary of part II and other views	

Part 3

16	<i>Non-Euclidean mechanics</i>	161
	Dostoevsky. Non-Euclidean mechanics	

<i>17 The question of absolute space</i>	163
Newton; Newtonian space. Relative motion. Magnetism and electricity. Ether drift?; absolute space? Einstein's idea. Kennedy-Thorndike experiment. The nature of space	
<i>18 Space, time, and space-time</i>	176
Space-time. Clocks and surveying. The invariance of distance. Change of axes. Invariance of the interval. Summary. Appendix on co-ordinate transformations. Exercises	
<i>19 Paradoxes of special relativity</i>	190
The 'paradoxes' of special relativity posed and solved	
<i>20 Gravitation and non-Euclidean geometry</i>	196
Gravity, its relative nature. The conventional element in measurement. The heated-plate and the cooled-plate universes, their connections with non-Euclidean geometries. The rubber-sheet model of gravity. Exercises	
<i>21 Speculations</i>	204
Gravitation in four-dimensional space-time, curvature, black holes. Speculations. Appendix, W. K. Clifford	
<i>22 Some last thoughts</i>	212
Meanings. Mathematical appendix on the connection between non-Euclidean geometry and special relativity, and on transformation groups	
<i>List of mathematicians and physicists</i>	217
<i>Bibliography</i>	219
<i>Index</i>	223