CONTENTS

1. THE COMPONENTS OF MECHANISM 1
1.1. Mechanism 1
1.2. Kinematic geometry 2
1.3. The rigid body 3
1.4. Joints 4
1.4.1. *Curve and straight-line contact; joint closure* 5
1.4.2. *Pairs; couplings* 5
1.4.3. *Lower pairs; linkages* 6
1.4.4. *Higher and lower pairs in practice* 11
1.5. Expansion of pairs; inversion of pairs and mechanisms 13
Examples 1A 16
1.6. Planar and spherical motion 17
1.7. The screw; general spatial movement 17
1.8. Substitute-linkages 20
Examples 1B 22
1.9. Inputs, outputs, actuator-pairs 22
1.10. Type-, number-, and size-synthesis 25
1.11. Symmetry, models, and practical design 27
Examples 1C 29

2. FREEDOM AND STRUCTURE IN MECHANISM 30
2.1. Freedom; connectivity; mobility 30
2.2. Freedom and constraint of a body 31
2.3. Criteria of mobility; their interpretation 33
2.4. Stationary configurations; uncertainty-configurations 37
2.5. Structure of kinematic chains; permutations 39
Examples 2A 41
2.6. Reciprocal pairs and connections 42
2.7. Analogies between statics and instantaneous kinematics 45
2.8. The wrench on a screw; the twist on a screw axis 47
Examples 2B 51

3. ELEMENTARY PLANAR AND SPATIAL DISPLACEMENTS 53
3.1. Introduction 53
3.2. Two locations of a lamina 53
3.3. Three locations of a lamina; centrodes 58
Examples 3A 63
3.4. The theorem of three centres 64
3.5. Two freedoms; systems of instantaneous centres 67
3.6. Location of instantaneous centres 70
3.7. Instantaneous centres and poles in synthesis; inversion 72
Examples 3B 75
3.8. Alt's diagram; crank-rocker synthesis; transmission angle 76
3.9. Transmission through profiles; pressure angle; engineering versus geometry 80
3.10. Grashof and non-Grashof linkages 81

Examples 3C 84
3.11. Elementary spatial displacements 85
3.12. The screw axis for a finite displacement 85
3.13. Axodes 88
3.14. The theorem of three instantaneous screw axes; the cylindroid 89
3.14.1. *Two parallel screws* 90
3.14.2. *Two general screws; the theorem of three screw axes; the cylindroid* 92
3.14.3. *The cylindroid; a simpler derivation* 96
3.14.4. *Using the theorem of three ISAs; the cylindroid represented by a circle* 98
Examples 3D 100

4. PLANAR ALGEBRAIC CURVES 102
4.1. Introduction 102
4.2. Algebraic curves; linkages; Kempe's theorem 102
4.3. Some transcendental curves 104
4.4. Non-existence of certain linkages 106
4.5. Rational and irrational curves 107
4.6. Order of a curve; Bézout's theorem 107
4.7. Homogeneous coordinates; the imaginary circular points 108
Examples 4A 110
4.8. Line-equations; class 110
4.9. Singularities; duality; Plücker's equations 112
4.10. Deficiency; degeneracy; tangents; asymptotes 115
Examples 4B 116
4.11. Freedom of a curve; some principles of body-guidance 117
4.12. Infinitesimal separations; osculation; curvature-matching 118
Examples 4C 120

5. INFINITESIMAL PLANAR KINEMATICS 122
5.1. Centrodes used to reproduce relative movement 122
5.2. Coordinates and conventions 123
5.3. The Euler–Savary equation 123
5.3.1. *The inflexion circle* 125
5.3.2. *Curvature of centrodes* 126
5.3.3. *Curvature of point-paths* 126
5.4. Envelopes of moving lines; cusp circle 127
5.5. Bobillier's theorem; Bobillier's constructions 129
5.5.1. *Aronhold's construction for path-curvature* 131
5.5.2. *Transitory substitute-linkages* 132
5.5.3. *Adding computational accuracy to graphical synthesis* 132
Examples 5A 133
5.6. Stationary curvature 135
5.7. The cubic of stationary curvature derived 136
5.7.1. *Properties of the cubic of stationary curvature* 138
5.7.2. *Graphical construction of the cubic of stationary curvature* 139
5.7.3. *Significance of μ and ν* 141
5.7.4. *Ball's point* 142

5.7.5. *The 'inverted' cubic, or pivot-point curve* 143
5.7.6. *Intersections of the two cubics* 145
Examples 5B 146
5.8. Instantaneous invariants 148
5.8.1. *Instantaneous centre and centrodes located* 150
5.8.2. *The Euler–Savary equation derived again* 152
5.8.3. *The cubic of stationary curvature derived again* 153
5.8.4. *The acceleration centre and the Bresse circle* 154
Example 5C 155
5.9. Introduction to infinitesimal Burmester theory 156

6. PLANAR DISPLACEMENTS THROUGH THREE AND MORE LOCATIONS 159
6.1. Introduction 159
6.2. Three locations of a lamina 159
6.3. Coordinated angular displacements 163
6.4. Structural error; Chebyshev spacing 165
6.5. Two-stage synthesis 167
Examples 6A 170
6.6. Four locations 171
6.7. The quadratic correspondence; the cubic curve of centre- and circle-points 172
6.8. The six poles for four locations 173
6.9. Opposite-pole quadrilaterals; the pole curve 174
6.10. Pole curve proved to be a cubic 176
6.11. Pole curve and centre-point curve coincide; properties; constructions 176
6.12. Four-location linkage synthesis; some difficulties of sequence and continuity 178
6.13. Circle-point curve 179
6.14. The four Burmester points 181
Examples 6B 182
6.15. Centre-point curve; its algebraic equation; infinitesimal convergence 183
6.16. The four-bar linkage function generator; algebraic synthesis 187
6.17. The four-bar linkage function generator; explicit angular relationships 191
6.18. Correction of errors 191

7. THE FOUR-BAR LINKAGE; COUPLER CURVES 194
7.1. The general nature and use of coupler curves 194
7.2. Transitional linkages 197
Examples 7A 198
7.3. The equation of the coupler curve 199
7.4. Multiple points on the coupler curve 201
7.5. Intersections with circle and line 204
7.6. The circle of singular foci; imaginary asymptotes 205
7.7. The Roberts–Chebyshev theorem; cognate linkages 205
7.8. Symmetrical coupler curves 208
7.9. Some other cognate linkages 210
7.10. Coupler curves from double-crank linkages 211

7.11. Cusps in coupler curves 212
7.12. Slider-crank coupler curve 213
7.13. Inverted slider-crank coupler curve 214
7.14. Coordination of a coupler-point's position with input angle 216
Examples 7B 218

8. THE GEOMETRICAL CAPABILITY OF PLANAR MECHANISMS 222
8.1. The four-bar linkage 222
8.2. The six-bar Watt-linkage; its extensions 224
8.3. The six-bar Stephenson-linkage with ternary link as base 225
8.4. The six-bar Stephenson-linkage with binary link as base 231
Examples 8A 234
8.5. Linkages augmented by gears and bands 235
8.6. Some special higher pairs, mainly profile-closed 239
Examples 8B 244

9. THREE-DIMENSIONAL GEOMETRY AND SPATIAL MECHANISM 246
9.1. Three dimensions and its Cartesian system 246
9.2. Algebraic surfaces and spatial curves; intersections; order 246
9.3. Algebraic spatial linkages 247
9.4. Quadric surfaces 249
9.5. Homogeneous coordinates; the imaginary spherical circle 249
9.6. Tangent-planes; class of a surface; duality; singularities 250
9.7. Class of a spatial curve; its normals 251
9.8. Ruled surfaces; reguli 251
9.8.1. *Properties of reguli; axial pencil of planes* 254
9.9. The degree of a ruled surface 256
9.10. Developables and cones 257
9.11. Curvature of surfaces and spatial curves 258
9.12. Striction; parameter of distribution; axodes 262
9.13. Rank; apparent cusps 266
Examples 9A 266
9.14. Coaxial helices of equal pitch 267
9.15. Two-way loci from simple lower-pair connections 270
Examples 9B 273

10. SOME SPATIAL MECHANISMS 275
10.1. Introduction 275
10.2. The *RSSR* linkage 276
10.3. The spherical four-bar linkage 279
10.4. The spatial *RSSP*, or *RS*(*RC*), linkage; the *PSSP* linkage 281
10.5. Bennett's linkage 284
Examples 10A 287
10.6. Some more linkages with input at an *RS*- link 288
10.7. Surface-intersections in a spatial loop 291
10.8. Spin-surfaces 293
10.9. Spatial coupler curves 296
10.10. Spatial linkages without *S*-pairs 297
10.11. Prospects for spatial mechanisms 301
Examples 10B 302

11. LINE GEOMETRY AND SPATIAL MECHANISM 304
11.1. Introduction 304
11.2. Plücker's line coordinates 304
Examples 11A 309
11.3. General complexes and congruences of lines 310
Examples 11B 313
11.4. The linear complex 314
11.4.1. *The general form of the linear complex* 315
11.4.2. *The linear complex; coaxial helices; reciprocal connections* 316
11.4.3. *The linear complex defined by five lines* 318
11.4.4. *Pairs of conjugates; their properties* 319
11.5. The linear congruence 320
11.5.1. *Forms of the linear congruence* 322
11.6. Linear dependence of lines 323
11.7. Higher complexes and congruences 326
Examples 11C 328

12. SCREW SYSTEMS 331
12.1. Reciprocal screws 331
12.2. The cylindroid; more properties 336
12.2.1. *A tangential planar section* 336
12.2.2. *A cone of the complex of normals to screws of the cylindroid* 336
12.2.3. *A line intersecting a screw orthogonally* 337
12.2.4. *Reguli of screws reciprocal to the screws on a cylindroid* 338
12.2.5. *The cylindroid in Plücker coordinates* 339
12.2.6. *The pitch-gradient; an infinitely long cylindroid* 340
Examples 12A 341
12.3. Screw systems; preamble to the survey 342
12.4. The one-system (one degree of freedom) 344
12.4.1. *The special one-system;* $h_\alpha = \infty$ 344
12.5. The two-system (two degrees of freedom) 344
12.5.1. *The first special two-system;* $h_\beta = h_\alpha$, *both finite* 345
12.5.2. *The second special two-system;* $h_\beta = \infty$ (*or* $-\infty$), h_α *finite* 345
12.5.3. *The third special two-system;* $h_\alpha = h_\beta = \infty$ 346
12.5.4. *The fourth special two-system;* $h_\alpha \to \infty$, $h_\beta = -k^2 h_\alpha$ 346
12.5.5. *The fifth special two-system;* $h_\alpha \to \infty$, $h_\beta = -k^2 h_\alpha$, $k = 1$ 346
12.6. The three-system; introduction 346
12.6.1. *The point common to the central planes of all cylindroids in the three-system* 347
12.6.2. *The concentric reguli* 347
12.6.3. *The pattern of concentric hyperboloids* 349
12.6.4. *The* (3, 2) *congruence of lines of a general three-system* 351
12.6.5. *The two singular planes; screws in parallel planes* 352
12.6.6. *The singular points Q′ and Q″* 353
12.6.7. *A pattern of* ∞^1 *hyperbolic paraboloids* 353

12.7. The three-system (three degrees of freedom) 356
12.7.1. *The first special three-system; $h_\beta = h_\alpha$, h_γ finite* 356
12.7.2. *The second special three-system; $h_\gamma = h_\beta = h_\alpha$, all finite* 356
12.7.3. *The third special three-system; $h_\alpha = \infty$, $h_\gamma \neq h_\beta$ (both finite)* 357
12.7.4. *The fourth special three-system; $h_\alpha = \infty$, $h_\beta = h_\gamma$ (both finite)* 357
12.7.5. *The fifth special three-system; $h_\alpha = h_\beta = \infty$, h_γ finite* 357
12.7.6. *The sixth special three-system; $h_\alpha = h_\beta = h_\gamma = \infty$* 357
12.7.7. *The seventh special three-system; $h_\alpha \to \infty$, $h_\gamma = -k^2 h_\alpha$, h_β finite* 358
12.7.8. *The eighth special three-system; $h_\alpha \to \infty$, $h_\gamma = -k^2 h_\alpha$, h_β finite, $k = 1$* 358
12.7.9. *The ninth special three-system; $h_\alpha \to \infty$, $h_\gamma = -k^2 h_\alpha$, $h_\beta = \infty$* 359
12.7.10. *The tenth special three-system; $h_\alpha \to \infty$, $h_\gamma = -k^2 h_\alpha$, $h_\beta = \infty$, $k = 1$* 359
Examples 12B 359
12.8. The four-system; introduction 361
12.8.1. *A pattern of ∞^2 hyperbolic paraboloids* 364
12.9. The four-system (four degrees of freedom) 365
12.9.1. *The first special four-system; $h'_\beta = h'_\alpha$, both finite* 366
12.9.2. *The second special four-system; $h'_\beta = \infty$, h'_α finite* 366
12.9.3. *The third special four-system; $h'_\beta = h'_\alpha = \infty$* 366
12.9.4. *The fourth special four-system; $h'_\alpha \to \infty$, $h'_\beta = -k^2 h'_\alpha$* 366
12.9.5. *The fifth special four-system; $h'_\alpha \to \infty$, $h'_\beta = -k^2 h'_\alpha$, $k = 1$* 367
12.10. The five-system (five degrees of freedom) 367
12.10.1. *The special five-system; $h'_\alpha = \infty$* 368
Examples 12C 368
12.11. Screw-systems summarized; curve- and surface-trajectories 369
12.12. Screw coordinates; linear dependence of screws and of freedoms 372

13. SCREW SYSTEMS APPLIED TO SPATIAL MECHANISMS 375
13.1 Introduction 375
13.2 Screw systems and the mobility of spatial linkages 375
Examples 13A 382
13.3. Stationary configurations of linkages 384
13.4. Uncertainty-configurations of linkages 389
Examples 13B 390
13.5. Screw systems and synthesis; shaft couplings 393
Examples 13C 401

14. BODY GUIDANCE IN THREE DIMENSIONS 403
14.1. Introduction 403
14.2. Spherical motion 404
14.3. Three, four, and five locations in spherical motion 405
14.4. The Euler–Savary equation in spherical motion 408
14.5. The cubic cone of stationary curvature in spherical motion 410
Examples 14A 412
14.6. General spatial motion 412
14.7. The cubic correspondence for four general locations 413
14.8. The cubic inflexion curve 414
14.9. Inflexion points for a general infinitesimal displacement 415
14.10. Points in a body whose successive positions are coplanar 416
14.11. Points in a body whose successive positions lie on circles 416
14.12. Points in a body whose successive positions lie on spheres 417
14.13. Practical body guidance by linkage-connections 418

15. MANIPULATOR-ARMS AND OTHER LINKAGE-CONNECTIONS 421
15.1. Introduction 421
15.2. Manipulator-arms 421
15.3. Algebraic connections; geometrical theory 427
15.3.1. *The feather of an algebraic connection* 428
15.3.2. *The invariance of feather of a* $\mathscr{C} = 1$ *connection* 429
15.3.3. *The feathers of two and three series-joined* $\mathscr{C} = 1$ *connections* 430
15.3.4. *Two theorems of Cayley* 431
15.3.5. *The elliptic trammel, an apparent exception* 434
15.3.6. *Reduced feathers* 435
15.3.7. *Algebraic cycloids* 437
15.3.8. *Further properties of algebraic connections* 440
15.4. Closure 440

REFERENCES 441

INDEX 457