

CONTENTS

	PAGE
PREFACE.	vii

PLANE TRIGONOMETRY

CHAPTER I

TRIGONOMETRIC FUNCTIONS OF AN ACUTE ANGLE

ART.		
1.	Introduction.	3
2.	Ratio.	4
3.	The Tangent, the Sine, and the Cosine	5
4.	The Cotangent, the Secant, and the Cosecant	9
5.	Trigonometric Functions of 45° , 30° , 60° , 0° , 90°	12
6.	Table of Values of Trigonometric Functions.	15
7.	Finding Heights and Distances by Means of Trigonometric Functions	16
8.	Solving Rectilinear Figures	20
9.	Miscellaneous Exercises.	23

CHAPTER II

FUNDAMENTAL RELATIONS AMONG THE TRIGONOMETRIC FUNCTIONS

10.	Introduction.	28
11.	Simple Relations.	28
12.	Identities and Conditional Equations.	30
13.	Relations Derived from the Pythagorean Theorem	32
14.	Verification of Identities	34
15.	Formulas from Right Triangle.	37
16.	Length of Line Segments	40
17.	Miscellaneous Exercises.	44

CHAPTER III

GENERAL DEFINITIONS OF TRIGONOMETRIC FUNCTIONS

18.	Definition of Angle.	48
19.	Rectangular Coordinates	49
20.	Definitions of the Trigonometric Functions of Any Angle	51
21.	Observations.	55
22.	Values of Trigonometric Functions for Special Angles.	55
23.	Fundamental Identities.	58
24.	Expressing a Trigonometric Function of Any Angle as a Function of an Acute Angle	58

ART.	PAGE
25. Functions of $\pm\theta$ and $180^\circ \pm \theta$ in Terms of Functions of θ	61
26. Miscellaneous Exercises.	64

CHAPTER IV

RIGHT TRIANGLES

27. Introduction.	67
28. Accuracy	67
29. Tables of Natural Trigonometric Functions	68
30. Solving Right Triangles.	70
31. Definitions	72
32. Solution of the Right Triangle by Slide Rule.	76
33. Slide-rule Solution of a Right Triangle When Two Legs Are Known	78
34. Table of Logarithms of Trigonometric Functions.	79
35. To Find the Logarithms of a Trigonometric Function of an Angle	79
36. To Find the Angle When the Logarithm Is Given	81
37. Solution of the Right Triangle by Means of Logarithms.	82
38. Solution of Rectilinear Figures.	85
39. Miscellaneous Exercises.	89

CHAPTER V

FORMULAS AND GRAPHS

40. Introduction.	94
41. The Radian.	94
42. Length of Circular Arc	96
43. Functions of $90^\circ - \theta$	99
44. Functions of $90^\circ + \theta$, $270^\circ + \theta$, $180^\circ \pm \theta$, $-\theta$	101
45. Functions of $(k 90^\circ \pm \theta)$	104
46. Graph of $y = \sin x$	106
47. Graph of $y = \cos x$	108
48. Graph of $y = \tan x$	109
49. Graphs of $y = \cot x$, $y = \sec x$, $y = \csc x$	111
50. Graphs and Periods of the Trigonometric Functions of $k\theta$	113
51. Miscellaneous Exercises.	116

CHAPTER VI

GENERAL FORMULAS

52. The Addition Formulas.	120
53. Proof of the Addition Formulas. Special Case.	120
54. Removal of Restrictions on the Addition Formulas.	124
55. Addition and Subtraction Formulas for the Tangent	125
56. The Double-angle Formulas and the Half-angle Formulas.	128
57. Conversion Formulas.	133
58. Miscellaneous Exercises.	136

CHAPTER VII

IMPORTANT FORMULAS RELATING TO TRIANGLES

59. Law of Sines.	140
60. Law of Tangents. Mollweide's Equations.	144

CONTENTS

xi

ART.	PAGE
61. Law of Cosines.	146
62. Miscellaneous Exercises.	149

CHAPTER VIII

OBLIQUE TRIANGLES

63. Introduction.	152
64. Form for Computation by Logarithms to Be Used in the Solution of Oblique Triangles	153
65. Case I. Given One Side and Two Angles.	153
66. Case II. Given Two Sides and the Angle Opposite One of Them	156
67. Case III. Given Two Sides and the Included Angle	161
68. The Half-angle Formulas	164
69. Case IV. Given Three Sides	167
70. Summary.	169
71. Miscellaneous	170

CHAPTER IX

INVERSE TRIGONOMETRIC FUNCTIONS

72. Inverse Trigonometric Functions.	177
73. Graphs of the Inverse Trigonometric Functions	178
74. Representation of the General Value of the Inverse Trigonometric Functions.	180
75. Principal Values	182
76. Relations among the Inverse Functions.	184
77. Examples Involving Inverse Trigonometric Functions.	185
78. Trigonometric Equations	189
79. Special Types of Trigonometric Equation	192
80. Equations Involving Inverse Functions.	195
81. Miscellaneous Exercises.	197

CHAPTER X

COMPLEX NUMBERS

82. Pure Imaginary Numbers.	200
83. Complex Numbers	201
84. Operations Involving Complex Numbers	201
85. Geometrical Representation of Complex Numbers	202
86. Polar Form of a Complex Number	203
87. Multiplication of Complex Numbers in Polar Form.	205
88. The Quotient of Two Complex Numbers in Polar Form.	206
89. Powers and Roots of Complex Numbers, De Moivre's Theorem.	208
90. Exponential Forms of a Complex Number.	211
91. The Hyperbolic Functions.	212
92. Miscellaneous Exercises.	214

CHAPTER XI

LOGARITHMS

93. Introduction.	216
94. Laws of Exponents.	216

ART.	PAGE
95. Definition of a Logarithm.	217
96. Laws of Logarithms.	218
97. Common Logarithms. Characteristic.	221
98. Effect of Changing the Decimal Point in a Number.	223
99. The Mantissa	224
100. To Find the Logarithm of a Number	224
101. Interpolation	225
102. To Find the Number Corresponding to a Given Logarithm	226
103. The Use of Logarithms in Computations	227
104. Cologarithms	229
105. Computation by Logarithms.	230
106. Suggestions on Computing by Logarithms.	231
107. Change of Base in Logarithms.	235
108. Solution of Equations of the Form $x = a^b$, $a = x^b$	235
109. Graph of $y = \log_{10} x$	237
110. Miscellaneous Exercises.	238

CHAPTER XII

THE SLIDE RULE

111. Introduction.	241
112. Reading the Scales.	241
113. Accuracy of the Slide Rule	243
114. Definitions	243
115. Multiplication.	244
116. Either Index May Be Used	245
117. Division.	246
118. Use of Scales <i>DF</i> and <i>CF</i> (Folded Scales)	247
119. The Proportion Principle	249
120. Use of the <i>CI</i> Scale.	250
121. Combined Multiplication and Division	251
122. Square Roots	253
123. Combined Operations Involving Square Roots.	254
124. The <i>S</i> (Sine) and <i>ST</i> (Sine Tangent) Scales	256
125. The <i>T</i> (Tangent) Scale	257
126. Combined Operations.	258
127. Solving a Triangle by Means of the Law of Sines.	259
128. To Solve a Right Triangle When Two Legs Are Given	262
129. To Solve a Triangle in Which Two Sides and the Included Angle Are Given.	263
130. To Solve a Triangle in Which Three Sides Are Given.	264
131. To Change Radians to Degrees or Degrees to Radians	265

SPHERICAL TRIGONOMETRY

CHAPTER XIII

THE RIGHT SPHERICAL TRIANGLE

132. Introduction.	269
133. The Spherical Triangle	269

ART.	PAGE
134. Formulas Relating to the Right Spherical Triangle.	271
135. Napier's Rules.	274
136. Two Important Rules.	277
137. Solution of Right Spherical Triangles.	278
138. The Ambiguous Case.	281
139. Polar Triangles	282
140. Quadrantal Triangles.	284
141. Miscellaneous	285

CHAPTER XIV

THE OBLIQUE SPHERICAL TRIANGLE

142. Law of Sines.	289
143. Law of Cosines for Sides	291
144. Law of Cosines for Angles.	293
145. The Six Cases	295
146. The Half-angle Formulas	295
147. Cases I and II. Given Three Sides or Given Three Angles	298
148. Napier's Analogies	300
149. Cases III and IV. Given Two Sides and the Included Angle or Given Two Angles and the Included Side	303
150. Cases V and VI. Two of the Given Parts Are Opposite. Double Solutions	304
151. Miscellaneous Exercises.	307

CHAPTER XV

VARIOUS METHODS OF SOLVING OBLIQUE SPHERICAL
TRIANGLES

152. Introduction.	309
153. Cases III and IV.	309
154. Observations and Illustrative Example	310
155. Case III. Alternate Method	312
156. Haversine Solution of Case III.	315
157. Cases V and VI	316
158. Observations and Illustrative Example	318
159. Cases I and II.	319
160. Miscellaneous Exercises.	320

CHAPTER XVI

APPLICATIONS

161. Nature of Applications	322
162. Definitions and Notations.	322
163. Course and Distance	323
164. The Celestial Sphere	328
165. The Astronomical Triangle	330
166. Given t, d, L ; to Find h and Z	331
167. To Find the Time and Amplitude of Sunrise.	333

	PAGE
ART.	
168. To Find the Time of Day.	335
169. Ecliptic. Equinoxes. Right Ascension. Sidereal Time	337
170. The Time Sight	340
171. Meridian Altitude. To Find the Latitude of a Place on the Earth	341
172. Given t, d, h , to Find L	343
173. Miscellaneous Exercises.	344
APPENDIX	351
INDEX.	365
ANSWERS	369