Contents

Principal Notations	xi
Chapter I	
Some Historical Background	1
1. The Parallel-Postulate Problem	1
2. The Lost Centuries	3
3. Saccheri and the "Near Miss"	5
4. The Correct Perspective—Gauss, Bolyai, Lobatchevsky	12
5. Not-Euclidean Geometry, Absolute Geometry	15
Chapter II	
Absolute Plane Geometry	19
1. Linear Sets and Linear Order	19
2. Half-Planes, Angles, and Angle Measure	27
3. Triangle Relations, Congruence, Foot in a Set	34
4. Non-intersecting Lines	40
5. Dedekind Cut, Continuity, A Basic Circle Property	45
6. Motions and Symmetries	50
Chapter III	
Hyperbolic Plane Geometry	58
1. Hyperbolic Parallels	58
2. Biangles, Hyperparallels	80
3. Saccheri and Lambert Quadrilaterals, Polygon Angle Sums	90
4. Angle of Parallelism Function, Triangle Defect, Distance Variations	107
5. The 3-Point Property, Cycles	126
6. Hyperbolic Compass and Straight Edge Constructions	157
7. Existence Problems; the Method of Associated Right Triangles	182

ix

Preface

Chapter IV	
A Euclidean Model of the Hyperbolic Plane	203
1. An Overview of the Model	203
2. Circular Inversions in E^2	211
3. Angle and Cross Ratio Invariance Under Inversion	222
4. Linear Order and Motions in the Model	243
5. Half-Planes, Angles and Angle Measure in the Model	261
6. Triangle Congruence in the Model, the Consistency of Hyperbolic Geometry	278
Appendix	
Distance Geometrics	291
Topic I. Metric Space and Metric Geometry	292
Topic II. A Spherical Metric	297
Topic III. Elliptic Geometry	308
Topic IV. Barbilian Geometries, the Cross Ratio Metric	319
Index	327