CONTENTS

FOREWORD —2
Acknowledgments; references and symbols

CHAPTER 0 GUIDE 1

CHAPTER 1 MAPS 4
Membership; maps; subsets and quotients; forwards and backwards; pairs; equivalences; products on a set; union and intersection; natural numbers; products on ω; Σ and Π; order properties of ω

CHAPTER 2 REAL AND COMPLEX NUMBERS 26
Groups; rings; polynomials; ordered rings; absolute value; the ring of integers; fields; the rational field; bounded subsets; the $\rightarrowtail$ notation; the real field; convergence; the complex field; the exponential maps

CHAPTER 3 LINEAR SPACES 53
Linear spaces; linear maps; linear sections; linear subspaces; linear injections and surjections; linear products; linear spaces of linear maps; bilinear maps; algebras; matrices; the algebras $^{s}\mathbf{K}$; one-sided ideals; modules

CHAPTER 4 AFFINE SPACES 74
Affine spaces; translations; affine maps; affine subspaces; affine subspaces of a linear space; lines in an affine space; convexity; affine products; comment

CHAPTER 5 QUOTIENT STRUCTURES 85
Linear quotients; quotient groups; ideals; exact sequences; diagram-chasing; the dual of an exact sequence; more diagram-chasing; sections of a linear surjection; analogues for group maps; group actions

CHAPTER 6 FINITE-DIMENSIONAL SPACES 100
Linear dependence; the basis theorem; rank; matrices; finite-dimensional algebras; minimal left ideals

CHAPTER 7 DETERMINANTS 116

Frames; elementary basic framings; permutations of n; the determinant; transposition; determinants of endomorphisms; the absolute determinant; applications; the sides of a hyperplane; orientation

CHAPTER 8 DIRECT SUM 132

Direct sum; $^2\mathbf{K}$-modules and maps; linear complements; complements and quotients; spaces of linear complements; Grassmannians

CHAPTER 9 ORTHOGONAL SPACES 146

Real orthogonal spaces; invertible elements; linear correlations; non-degenerate spaces; orthogonal maps; adjoints; examples of adjoints; orthogonal annihilators; the basis theorem; reflections; signature; Witt decompositions; neutral spaces; positive-definite spaces; euclidean spaces; spheres; complex orthogonal spaces

CHAPTER 10 QUATERNIONS 174

The algebra $\mathbf{H}$; automorphisms and anti-automorphisms of $\mathbf{H}$; rotations of $\mathbf{R}^4$; linear spaces over $\mathbf{H}$; tensor product of algebras; automorphisms and anti-automorphisms of $^4\mathbf{K}$

CHAPTER 11 CORRELATIONS 198

Semi-linear maps; correlations; equivalent correlations; algebra anti-involutions; correlated spaces; detailed classification theorems; positive-definite spaces; particular adjoint anti-involutions; groups of correlated automorphisms

CHAPTER 12 QUADRIC GRASSMANNIANS 223

Grassmannians; quadric Grassmannians; affine quadrics; real affine quadrics; charts on quadric Grassmannians; Grassmannians as coset spaces; quadric Grassmannians as coset spaces; Cayley charts; Grassmannians as quadric Grassmannians; further coset space representations

CHAPTER 13 CLIFFORD ALGEBRAS 240

Orthonormal subsets; the dimension of a Clifford algebra; universal Clifford algebras; construction of the algebras; complex Clifford algebras; involuted fields; involutions and anti-involutions; the Clifford group; the uses of conjugation; the map N; the Pfaffian chart; Spin groups; The Radon–Hurwitz numbers

CHAPTER 14 THE CAYLEY ALGEBRA 277

Real division algebras; alternative division algebras; the Cayley algebra; Hamilton triangles; Cayley triangles; further results; the Cayley projective line and plane

CHAPTER 15 NORMED LINEAR SPACES 288

Norms; open and closed balls; open and closed sets; continuity; complete normed affine spaces; equivalence of norms; the norm of a continuous linear map; continuous bilinear maps; inversion

CHAPTER 16 TOPOLOGICAL SPACES 311

Topologies; continuity; subspaces and quotient spaces; closed sets; limits; covers; compact spaces; Hausdorff spaces; open, closed and compact maps; product topology; connectedness

CHAPTER 17 TOPOLOGICAL GROUPS AND MANIFOLDS 336

Topological groups; homogeneous spaces; topological manifolds; Grassmannians; quadric Grassmannians; invariance of domain

CHAPTER 18 AFFINE APPROXIMATION 353

Tangency; differentiable maps; complex differentiable maps; properties of differentials; singularities of a map

CHAPTER 19 THE INVERSE FUNCTION THEOREM 375

The increment formula; the inverse function theorem; the implicit function theorem; smooth subsets; local maxima and minima; the rank theorem; the fundamental theorem of algebra; higher differentials

CHAPTER 20 SMOOTH MANIFOLDS 399

Smooth manifolds and maps; submanifolds and products of manifolds; dimension; tangent bundles and maps; particular tangent spaces; smooth embeddings and projections; embeddings of projective planes; tangent vector fields; Lie groups; Lie algebras

CHAPTER 21 TRIALITY 435

Transitive actions on spheres; the first few diagrams; getting further; the companion involution; the triality automorphism; the group G_2; the action of Spin 9 on S^{15}; the action of Spin 10 on S^{31}; G_2 as a Lie group; other aspects of triality; quadric triality

BIBLIOGRAPHY 463 LIST OF SYMBOLS 467 INDEX 475