CONTENTS

PREFACE v

SYMBOLS vi

CHAPTER I. RECURSIVE CONVERGENCE 1

Primitive and general recursive functions
Recursive arithmetic and its extensions
Recursive convergence and relative convergence
The reduced sequence
Recursive limits and tests for recursive convergence
Primitive and general recursive real numbers

CHAPTER II. RECURSIVE AND RELATIVE CONTINUITY 39

Uniform and doubly uniform equivalents of a relatively continuous recursive function
Upper and lower bounds and the impossibility of a proof of their attainment
Conditions which ensure that a relatively continuous function vanishes if it changes sign, and the non-existence of a recursive root in the general case

CHAPTER III. RECURSIVE AND RELATIVE DIFFERENTIABILITY . . 57

The mean value inequalities
Doubly uniform equivalent of a relatively differentiable function
The mean value theorem
Taylor's theorem
The uniform mean value theorem
The existence of relatively differentiable functions not satisfying the uniform mean value theorem

CHAPTER IV. THE RELATIVE INTEGRAL 87

Ruled functions
Relatively integrable functions
Darboux's theorem
Continuity of, and derivative of, the relative integral
Substitution in the relative integral

CHAPTER V. THE ELEMENTARY FUNCTIONS 96

The relatively exponential, logarithmic and circular functions
Addition formulae, relative periodicity
Inverse functions

CHAPTER VI. TRANSFINITE ORDINALS 106

The relation of transfinite ordinals to the representation of numbers in a scale
The decreasing ordinal theorem
The use of majorant variables and a generalisation of scale representation
The generalised ordinal theorem

APPENDIX. RECURSIVE IRRATIONALITY AND TRANSCENDENCE . . . 123

Primitive and general recursive irrationality and transcendence
The primitive recursive transcendence of e and π

INDEX 137